MATHEMATICS-IX

Module - 1

INDEX

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NUMBER SYSTEMS

BASIC CONCEPTS AND IMPORTANT RESULTS

1. Natural Numbers (N) :

Counting numbers are known as natural numbers. Thus 1, 2, 3, 4, ...etc. are natural numbers.
- ✯ The first and the least natural number is 1 (one)
- ✯ Consecutive natural nos. differ by 1 (one).

2. Whole numbers (w) :

All natural numbers together with '0' form whole numbers. Thus 0, 1, 2, 3, 4, ... etc. one are whole nos.
- ✯ The first and the least whole number is zero.
- ✯ Consecutive whole number differe by one.

3. Integers (I or Z) :

All natural nos. 0 and negative of natural nos. form integers for example. ......−4, −3, −2, −1, 0, 1, 2, 3, 4, ... etc.
- ✯ O is neither a negative nor a positive number. It is a neutral no.

4. Prime numbers (P) :

A natural number, which is greater than 1 and divisible by one and by itself only, is called a prime number. For eg : 2, 3, 5, 7, 11, ...
- ✯ The smallest prime number is 2
- ✯ Except 2 ; all other prime nos. are odd.

5. Composite number (C) :

A natural number, which is greater than 1 and is not prime, is called a composite number. Thus 4, 6, 8, 9, 10, 12, 14, ...
- ✯ The smallest composite number is 4.
- ✯ A composite number can be even or odd.
- ✯ It has atleast three distinct factor.

6. Co-prime numbers :

If two numbers do not have any factor (other than 1) common; the numbers are said to be co-prime Thus (i) 6 and 25 are coprime, no any common factor other than 1. (ii) 3 and 5 are co-prime, no any common factor other than 1.
- ✯ It is not necessary that any of the two co-prime numbers has to be prime also.
- ✯ All consecutive nos. are coprime.

7. Terminating decimals :

The decimal expansion ends after a finite number of steps of division. Such decimal expansions are called terminating decimals

For example : $\frac{2}{5} = 0.4$, $\frac{33}{8} = 4.125$ and so on.

8. Non-terminating decimals :

The decimal expansions never come to an end. Such decimal expansions are called non-terminating

For example = $\frac{2}{11} = 0.1818...$, $\frac{16}{45} = 0.3555......$

9. Rational Numbers (Q) :

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are known as rational numbers.

or

A number is rational if and only if its decimal representation is terminating or non-terminating but recurring

Ex. $\frac{2}{5}$, 3, $\frac{5}{1}$, 1.75, 1.666......, 4.23535, ..., $\frac{7}{9}$

10. Irrational numbers :

A number which cannot be put in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called an irrational number

or

A number whose decimal expression is non-terminating and non recurring is called an irrational number.

Eg : $\sqrt{5}$, $\sqrt{3}$, $5\sqrt{7}$, $\sqrt{3}+2$, $\frac{1}{3+\sqrt{7}}$, π, $\sqrt[4]{3}$, ...

11. Non-terminating : Repeating (or Recurring) decimals :

A decimal in which a digit or a group of digits repeats continually or periodically is called a repeating or a recurring or a periodic decimal.

Ex : $\frac{5}{6} = 0.8333... = 0.8\overline{3}$; $\frac{2}{11} = 0.181818...... = 0.\overline{18}$

☆ Put a bar ($\overline{\ }$) above those digit/digits which are repeated.

12. Real Numbers (R) :

Rational numbers and irrational numbers taken together form real numbers.

13. Pure recrring decimal :

It is a decimal representation in which all the digits after the decimal point are repeated Eg: $2.\overline{53}$, $0.\overline{35}$, $0.\overline{315}$, ...

14. Mixed recurring decimal :

It is a decimal representation in which there are one or more digits present before the repeating digits.

Eg : $0.3\overline{2}$, $1.2\overline{3}$, $35.1\overline{23}$, ...

15. Negative of an irrational number is an irrational number.

16. The sum or difference of a rational number and an irrational number is an irrational number.

17. The product of a non-zero rational number and an irrational number is an irrational number.

18. The sum, difference, product and quotient of two irrational numbers need not be an irrational number.

19. There are an infinite number of rational (irrational) numbers between two rational (or irrational) numbers.

20. If a is a rational number and n is a positive integer such that the n^{th} root of a is an irrational number, then $a^{1/n}$ is called a surd eg. $\sqrt{7}$, $\sqrt{3}$, $\sqrt{11}$ etc

21. If $\sqrt[n]{a}$ is a surd, or radical then 'n' is known as ordern or index of surd and 'a' is known as radicand.

22. A surd which has unity only as rational factor is called a pure surd.

Eg. $\sqrt{5}$, $\sqrt{11}$, $\sqrt{7}$, $\sqrt{335}$, ...

23. A surd which has a rational factor other than unity is called a mixed surd.

Eg . $2\sqrt{5}$, $3\sqrt{11}$, ...

24. Surds having same irrational factors are called similar or like surds.

25. Only similar surds can be added or subtracted by adding or subtracting their rational parts.

26. Surds of same order can be multiplied or divided.

27. If the surds to be multiplied or to be divided are not of the same order, we first reduce them to the same order and then multiply or divide.

28. The two irrational numbers whose product is a rational number, are called rationalising factor of each other. For eg : $x - \sqrt{y}$ is called rationalising factor $x + \sqrt{y}$.

Similarly $\sqrt{3}$ is a R.F. of $6\sqrt{3}$ Similarly $5^{\frac{1}{3}}$ is a R.F. of $5^{\frac{2}{3}}$

29. The surds which differ only in sign (+ or −) between the terms connecting them, are called conjugate surds eg. $\sqrt{5}+\sqrt{3}$ and $\sqrt{5}-\sqrt{3}$ or $2+\sqrt{5}$ and $2-\sqrt{5}$ are conjugate surds (binomial).

✯ Sum and product of two cojugate binomial factors are always rational numbers.

30. Laws of exponents for Real numbers :

(i) $a^m \times a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $\frac{a^m}{a^n} = a^{m-n}$; $m > n$

(iv) $a^m \times b^m = (a \times b)^m$ (v) $a^{-m} = \frac{1}{a^m}$ or $\frac{1}{a^{-m}} = a^m$, if $a \neq 0$

(vi) $(a \times b)^m = a^m \times b^m$ (vii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (viii) $a^0 = 1$ where a is any rational no.

(ix) $(1)^p = 1$ where p is any rational no.

(x) If $a \neq 1$ and $a^p = a^q$ then $p = q$ where p & q are rational nos

(xi) $\sqrt{a} = a^{\frac{1}{2}}$, $\sqrt[3]{a} = a^{\frac{1}{3}}$ and $\sqrt[n]{a} = a^{\frac{1}{n}}$

(xii) $(-a)^m = a^m$, if m is even and $(-a)^m = -a^m$, if m is odd.

31. Laws of radicals :

(i) $\left(\sqrt[n]{a}\right)^n = a$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ (iii) $\frac{\sqrt[n]{a}}{\sqrt[n]{a}} = \sqrt[n]{\frac{a}{b}}$

(iv) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (v) $\frac{\sqrt[p]{a^n}}{\sqrt[p]{a^m}} = \sqrt[p]{a^{n-m}}$ (vi) $\sqrt[p]{a^n \times a^m} = \sqrt[p]{a^{n\ m}}$

(vii) $\sqrt[p]{(a^n)^m} = \sqrt[p]{a^{nm}}$

32. Indentities related to square roots :

(i) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ (ii) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$ (iv) $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right) = a^2 - \left(\sqrt{b}\right)^2 = a^2 - b$

(v) $\left(\sqrt{a}+b\right)\left(\sqrt{a}-b\right) = a - b^2$ (vi) $\left(\sqrt{a}+\sqrt{b}\right)^2 = a + 2\sqrt{ab} + b$

(vii) $\left(\sqrt{a}-\sqrt{b}\right)^2 = a - 2\sqrt{ab} + b$ (viii) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}+\sqrt{d}\right) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

SOLVED PROBLEMS

Ex.1 Is zero a rational number ? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number. We can write zero in the form $\frac{p}{q}$ whose p and q are integers and $q \neq 0$.

so, 0 can be written as

$\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc

Ex.2 Find six rational numbers between 3 and 4.

Sol. **Hint :** first rational number between 3 and 4

$= \frac{3+4}{2} = \frac{7}{2}$

Ex.3 Find five rational numbers between $\frac{3}{5}$ **and** $\frac{4}{5}$**.**

Sol. **Hint :** Let $a = \frac{3}{5}$, $b = \frac{4}{5}$, $n = 5$

$$d = \frac{b-a}{n+1} = \frac{\frac{4}{5} - \frac{3}{5}}{5+1} = \frac{1}{30}$$

so, Rational number are

a + d, a + 2d, a + 3d........

Ex.4 State whether the following statements are true or false ? Give reasons for you answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True, the collection of whole number contain all natural number.

(ii) False, −2 is not whole number

(iii) False, $\frac{1}{2}$ is a rational number but not whole number.

Ex.5 State whether the following statements are true or false ? Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Sol. (i) True, since collection of real number consist of rational and irrational.

(ii) False, because no negative number can be the square root of any natural number.

(iii) False, 2 is real but not irrational.

Ex.6 Are the square roots of all positive integers irrational ? If not, give an example of the square root of a number that is a rational number.

Sol. No, $\sqrt{4} = 2$ is a rational number.

Ex.7 Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Sol. (i) $\frac{36}{100} = 0.36$(Terminating)

(ii) $\frac{1}{11} = 0.090909$.......(Non terminating Repeating)

$$\begin{array}{r|l} 11 & 1.00000 \; (0.090909.... \\ & -99 \\ \hline & 100 \\ & 99 \\ \hline & 100 \\ & 99 \\ \hline & 1 \end{array}$$

(iii) $4\frac{1}{8} = \frac{33}{8}$ = 4.125 (terminating decimal)

(iv) $\frac{3}{13}$ = 0.230769230769......

= $0.\overline{230769}$ (Non Terminating repeating)

(v) $\frac{2}{11} = 0.1818....... = 0.\overline{18}$

(Non Terminating repeating)

(vi) $\frac{329}{400}$ = 0.8225 terminating

Ex.8 Classify the following numbers as rational or irrational :

(i) $2 - \sqrt{5}$ (ii) $(3 + \sqrt{23}) - \sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Sol. (i) $\because$ 2 is a rational number and $\sqrt{5}$ is an irrational number

$\therefore$ $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

$\Rightarrow$ $(3 + \sqrt{23}) - \sqrt{23}$ = 3 is a rational number.

(Rest Try Yourself)

Ex.9 Simplify each of the following expressions

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$ (ii) $(3 + \sqrt{3})(3 - \sqrt{3})$ (iii) $(\sqrt{5} + \sqrt{2})^2$ (iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Sol. (i) $(3 + \sqrt{3})(2 + \sqrt{2}) = 3(2 + \sqrt{2}) + \sqrt{3}(2 + \sqrt{2})$

= $6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2$ = 9 – 3 = 6

(Rest Try Yourself)

Ex.10 Recall, p is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol.. $\frac{c}{d} = \frac{22}{7}$ which is approximate value of π

Ex.11 Rationalise the denominators of the following

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Sol. (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6}$

$$= \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$$

(Rest Try Yourself)

Ex.12 Find :
(i) $(64)^{1/2}$ (ii) $32^{1/5}$ (iii) $125^{1/3}$

Sol.. (i) $(64)^{1/2} = (8^2)^{1/2} = (8^{2\times\frac{1}{2}}) = 8^1 = 8$

(ii) $32^{1/5} = (2^5)^{1/5} = (2^{5\times\frac{1}{5}}) = 2^1 = 2$

(Rest Try Yourself)

Ex.13 Find :
(i) $9^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$ (iv) $125^{-1/3}$

Sol.. (i) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = (3)^3 = 27$

(ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

(Rest Try Yourself)

Ex.14 Simplify :

(i) $2^{2/3}.2^{1/5}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{1/2}}{11^{1/4}}$ (iv) $7^{1/2} \,.\, 8^{1/2}$

Sol. (i) $2^{\frac{2}{3}} \,.\, 2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} = 3^{-21}$

(Rest Try Yourself)

Ex.15 Insert 4 rational numbers between $\frac{2}{3}$ and $\frac{5}{3}$.

Sol. As numbers to be inserted are more than 3, we would follow method II., (Method I, a < $\frac{a+b}{2}$ < b)

Here the numbers given are $\frac{2}{3}$ and $\frac{5}{3}$ both of which have the same denominator.

$\therefore$ We multiply numerator and denominator of each number by (4 + 1) = 5

to get $\frac{2\times5}{3\times5}$ and $\frac{5\times5}{3\times5}$ or $\frac{10}{15}$ and $\frac{25}{15}$. Any 5 integers between 10 and 25 are 11, 12, 13, 14, 15.

$\therefore$ Required rational numbers between the two given numbers are $\frac{11}{15}, \frac{12}{15}, \frac{13}{15}, \frac{14}{15}, \frac{15}{15}$.

Ex.16 Convert $\frac{237}{16}$ in the decimal form.

Sol.

$$\begin{array}{r|l} 16 & 237\ (14.8125 \\ & \underline{16} \\ & 77 \\ & \underline{64} \\ & 130 \\ & \underline{128} \\ & 20 \\ & \underline{16} \\ & 40 \\ & \underline{32} \\ & 80 \\ & \underline{80} \\ & \times \end{array}$$

$\therefore \frac{237}{16} - 14.8125$

Ex.17 Convert $0.7\overline{283}$ into the form $\frac{p}{q}$.

Sol. The given number is $0.7\overline{283} = 0.7283283 \ldots$.

Let, $x = 0.7283283 \ldots$ ...(1)

Here after decimal there is only one digit namely 7, which is not recurring.

$\therefore$ We multiply both sides of equation (1) by 10 to get $10x = 7.283283\ldots$ ...(2)

Now after decimal 3 digits are recurring (283).

$\therefore$ We multiply both sides of equation (2) by 1000 to get, $10000x = 7283.283\ldots$ ...(3)

Subtracting equation (2) from equation (3), we get $90x = 7276$

$\Rightarrow x = \frac{7276}{9990} = \frac{3638}{4995}$ which is the required form of the number.

Ex.18 Write 3 irrational number between 4.75 and 4.76.

Sol. Keeping in mind that decimal representation of an irrational number is neither terminating nor recurring, we can write any three numbers between 4.75 and 4.76 whose decimal representation is neither terminating nor recurring e.g., 4.7513428965832..., 4.7523471098623..., 4.7534829153785... .

Ex.19 Locate $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ on number line.

Sol. We know that $5 = 2^2 + 1^2$. So on real number line X'OX, take a point A so that OA = 2 units. At A, draw a ray AY_1 perpendicular to real number line. Now with A as centre and 1 unit as radius draw an arc intersecting ray AY_1 at B_1. Join OB_1. With O as centre and OB_1 as radius draw an arc intersecting number line at P_1. P_1 is the point on number line representing $\sqrt{5}$ i.e., $OP_1 = \sqrt{5}$.

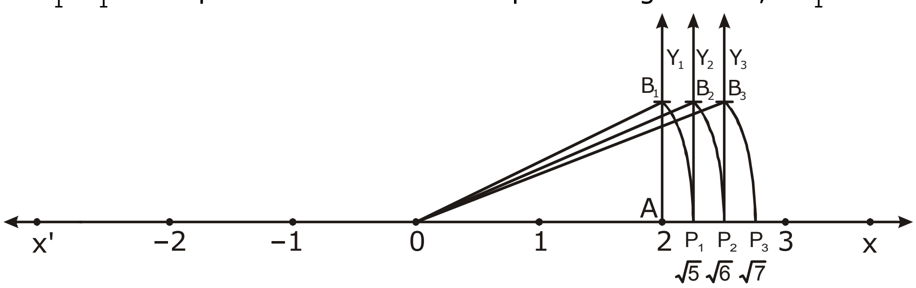


Fig. 11 Representing $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ on number line.

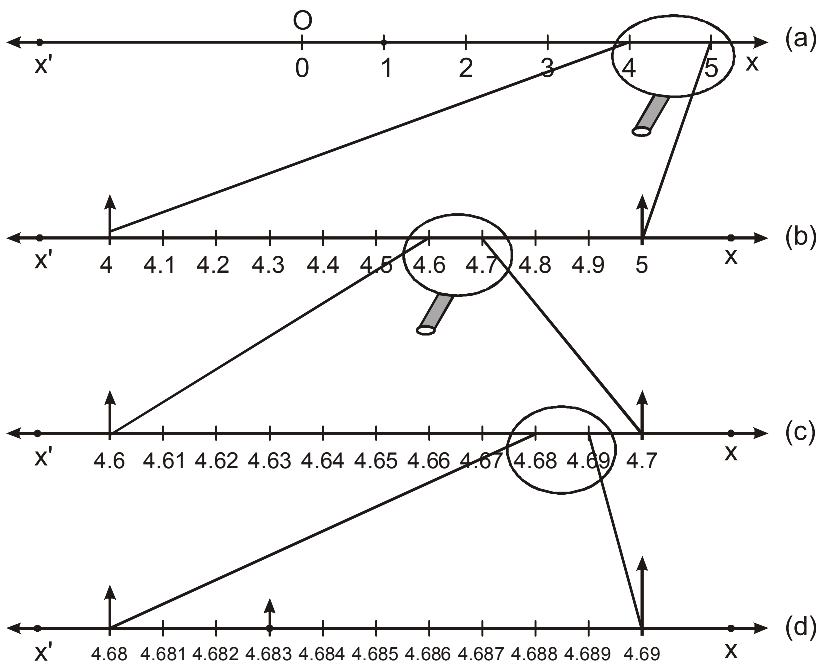
Now at P_1 draw ray P_1Y_2 perpendicular to number line and with P_1 as centre and 1 unit as radius draw an arc intersecting P_1Y_2 at B_2. Join OB_2. With O as centre and OB_2 as radius draw an arc intersecting the number line at P_2. P_2 is the point representing the location of $\sqrt{6}$. Again at P_2 draw a ray P_2Y_3 perpendicular to number line and cut an arc at B_3 on it with arc radius 1 unit and centre as P_2. Join OB_3. With O as centre and OB_3 as radius draw another arc intersecting the number line at P_3. P_3 is the point corresponding to $\sqrt{7}$.

Ex.20 With the help of examples show that the quotient of two irrational numbers can be rational or irrational.

Sol. Consider two irrational numbers a = $3\sqrt{2}$ and b = $5\sqrt{2}$ then their quotient $\frac{a}{b} = \frac{3\sqrt{2}}{5\sqrt{2}} = \frac{3}{5}$ which is rational, while if we take two numbers as c = $3\sqrt{6}$ and d = $\sqrt{8}$ both of which are irrational then their quotient $\frac{c}{d} = \frac{3\sqrt{6}}{\sqrt{8}} = \frac{3\sqrt{6}}{2\sqrt{2}} = \frac{3}{2} \times \sqrt{\frac{6}{2}} = \frac{3\sqrt{3}}{2}$ which is an irrational number.

Ex.21 Locate 4.683 on number line by the method of successive magnification.

Sol. Lie between 4–5, 4.6–4.7, 4.68–4.69.



Visualization of 4.683 on number line.

Ex.22 If $\frac{-32 \times 2^{x+5} + (2^x)^2}{2 \times 2^{x+1} - 2^{12}} = 2^{3x-10}$. Find the value of x, given that $x \neq 10$.

Sol. $\frac{(2^x)^2 - 32 \times 2^{x+5}}{2 \times 2^{x+1} - 2^{12}} = 2^{3x-10} \Rightarrow \frac{2^{x \times 2} - 2^5 \times 2^{x+5}}{2^{1+x+1} - 2^{12}} = 2^{3x-10} \Rightarrow \frac{2^{2x} - 2^{5+x+5}}{2^{2+x} - 2^{12}} = 2^{3x-10} \Rightarrow \frac{2^{x+x} - 2^{x+10}}{2^{x+2} - 2^{10+2}} = 2^{3x-10}$

$\Rightarrow \frac{2^x \cdot 2^x - 2^x \cdot 2^{10}}{2^x \cdot 2^2 - 2^{10} \cdot 2^2} = 2^{3x-10} \quad \Rightarrow \frac{2^x(2^x - 2^{10})}{2^2(2^x - 2^{10})} = 2^{3x-10} \quad \Rightarrow \frac{2^x}{2^2} = 2^{3x-10}$

$\Rightarrow 2^{x-2} = 2^{3x-10} \quad \Rightarrow x - 2 = 3x - 10 \quad \Rightarrow 2x = 8$

$\Rightarrow x = 4.$

Ex.23 If $2^x = 5^y = 10^z$, then prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.

Sol. Let $2^x = 5^y = 10^z = K$. $\Rightarrow$ $2 = K^{1/x}, 5 = K^{1/y}, 10 = K^{1/z}$

Now we know that $2 \times 5 = 10$ $\Rightarrow$ $K^{\frac{1}{x}} \times K^{\frac{1}{y}} = K^{\frac{1}{z}}$ $\Rightarrow$ $K^{\frac{1}{x}+\frac{1}{y}} = K^{\frac{1}{z}}$

$\Rightarrow$ $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.

Ex.24 If $x = \sqrt{3} + 1$, find the value of $\left(x+\frac{2}{x}\right)^2$.

Sol. $x = \sqrt{3}+1$ $\Rightarrow$ $\frac{2}{x}=\frac{2}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{2(\sqrt{3}-1)}{(\sqrt{3})^2-1^2}$

$=\frac{2(\sqrt{3}-1)}{3-1}=\frac{2(\sqrt{3}-1)}{2}=\sqrt{3}-1$ $\therefore$ $\left(x+\frac{2}{x}\right)^2=(\sqrt{3}+1+\sqrt{3}-1)^2=(2\sqrt{3})^2=4\times(\sqrt{3})^2$ $= 4 \times 3 = 12$.

Ex.25 If $x = 2 + \sqrt{3}$, find the value of $x^2 + \frac{1}{x^2}$.

Sol. $x = 2 + \sqrt{3}$ $\Rightarrow$ $\frac{1}{x}=\frac{1}{2+\sqrt{3}}\times\frac{(2-\sqrt{3})}{(2-\sqrt{3})}=\frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2}$ $\therefore$ $\frac{1}{x}=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$

Also $\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2$ $\therefore$ $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$ $=(2+\sqrt{3}+2-\sqrt{3})^2-2$ $= 4^2 - 2 = 16 - 2 = 14$.

Ex.26 If $x = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $y = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, find the value of $3x^2 + 4xy + 3y^2$.

Sol. $x=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}=\frac{(\sqrt{5})^2+(\sqrt{2})^2+2\sqrt{5}\times\sqrt{2}}{5-2}=\frac{5+2+2\sqrt{10}}{3}=\frac{7+2\sqrt{10}}{3}$

$y=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}=\frac{(\sqrt{5})^2+(\sqrt{2})^2-2\sqrt{5}\sqrt{2}}{5-2}=\frac{5+2-2\sqrt{10}}{3}=\frac{7-2\sqrt{10}}{3}$

$\therefore$ $x + y = \frac{7+2\sqrt{10}}{3}+\frac{7-2\sqrt{10}}{3}=\frac{7+2\sqrt{10}+7-2\sqrt{10}}{3}=\frac{14}{3}$ Also, $xy = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}=1$

Hence $3x^2 + 4xy + 3y^2 = 3(x^2 + y^2) + 4xy = 3\left[(x+y)^2-2xy\right]+xy$

$=3\left[\left(\frac{14}{3}\right)^2-2.1\right]+4.1=3\left[\frac{196}{9}-2\right]+4=3\left[\frac{196-18}{9}\right]+4=\frac{178}{3}+4=\frac{178+12}{3}=\frac{190}{3}$

Ex.27 If $x = \frac{1}{\sqrt{5}+2}$, find the value of $x^2 + 4x - 1$ and $x^3 - 2x^2 - 25x + 7$.

Sol. $x = \frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}=\frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}=\frac{\sqrt{5}-2}{5-4}=\sqrt{5}-2$ $\Rightarrow$ $x + 2 = \sqrt{5}$ $\Rightarrow$ $(x + 2)^2 = (\sqrt{5})^2 \Rightarrow x^2 + 4x + 4 = 5$

$\Rightarrow$ $x^2 + 4x - 1 = 0$ Also $x^3 - 2x^2 - 25x + 7 = (x^2 + 4x - 1)(x - 6) + 1$

(Here we observe that if $(x^3 - 2x^2 - 25x + 7)$ is divided by $x^2 + 4x - 1$, quotient is $x - 6$ and remainder $= 1$. So we can use dividend = divisor × quotient + remainder, to get the above relationship.)

$\therefore$ $x^3 - 2x^2 - 25x + 7 = 0 \times (x - 6) + 1 = 1$.

EXERCISE – I | UNSOLVED PROBLEMS

Q.1 Find 3 rational number between 2 and 5.

Q.2 Find 4 rational numbers between 4 and 5.

Q.3 Find three rational number between $\frac{6}{5}, \frac{7}{5}$

Q.4 Express $\frac{7}{8}$ in the decimal form by long division method.

Q.5 Convert $\frac{35}{16}$ into decimal form by long division method.

Q.6 Find the decimal representation of $\frac{8}{3}$.

Q.7 Express $\frac{2}{11}$ as a decimal fraction.

Q.8 Represent $\frac{1}{2}$ and $-\frac{1}{2}$ on the number line.

Q.9 Represent $\frac{4}{7}$ on number line.

Q.10 Represent $\frac{-9}{5}$ on number line.

Q.11 Express each of the following numbers in the form $\frac{p}{q}$.

(i) 0.15 (ii) 0.675 (iii) –25.6875

Q.12 Express each of the following decimals in the form $\frac{p}{q}$.

(i) $0.\overline{6}$ (ii) $0.\overline{35}$ (iii) $0.\overline{585}$

Q.13 Convert the following decimal numbers in form $\frac{p}{q}$

(i) $5.\overline{2}$ (ii) $23.\overline{43}$

Q.14 If $\frac{1}{7} = 0.\overline{142857},$ write the decimal expression of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}$, and $\frac{5}{7}$ without actually doing the long division.

Q.15 Express the following decimals in the form $\frac{p}{q}$

(i) $0.3\overline{2}$ (ii) $0.12\overline{3}$

Q.16 Insert a rational and an irrational number between 2 and 3.

Q.17 Find two irrational numbers between 2 and 2.5.

Q.18 Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Q.19 Find two irrational numbers between 0.12 and 0.13.

Q.20 Find two rational numbers between 0.232332333233332... and 0.252552555255552.....

Q.21 Find a rational number and also an irrational number between the numbers a and b given below :
a = 0.101001000100001..... ,
b = 0.1001000100001.....

Q.22 Find one irrationl number between the number a and b given below :

a = 0.1111..... = $0.\overline{1}$ and b = 0.1101

Q.23 Examine, whether the following numbers are rational or irrational :

(i) $(\sqrt{2}+2)^2$ (ii) $(5+\sqrt{5})(5-\sqrt{5})$

Q.24 State giving reasons, whether each one of the following number is rational or irrational

(i) $-\sqrt{5}$ (ii) $2+\sqrt{6}$ (iii) $5\sqrt{3}$

(iv) $(\sqrt{7}-2)$ (v) $\frac{7}{3\sqrt{5}}$ (vi) $(3+\sqrt{3})^2$

Q.25 Represent $\sqrt{3.28}$ geometrically on the number line.

Q.26 Evaluate each of the following :-

(i) $2^5 \times 5^2$ (ii) $(2^3)^2$ (iii) $\left(\frac{7}{9}\right)^3$

(iv) $\left(\frac{2}{5}\right)^{-3}$ (v) $\left(\frac{4}{5}\right)^7 \div \left(\frac{5}{4}\right)^{-5}$

Q.27 Evaluate the following :-

(i) $(216)^{\frac{-2}{3}}$ (ii) $\left(\frac{121}{169}\right)^{\frac{-3}{2}}$ (iii) $\left(\sqrt{81}\right)^{\frac{-3}{4}}$

(iv) $\left(\sqrt[3]{64}\right)^{\frac{-1}{2}}$ (v) $\left(\sqrt{25}\right)^{-7} \times \left(\sqrt{5}\right)^{-5}$

(vi) $\left(\frac{5^{-1} \times 7^{2}}{5^{2} \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^{3}}{5^{3} \times 7^{-5}}\right)^{\frac{-5}{2}}$

Q.28 Simplify the following :-

(i) $\sqrt[3]{ab^2} \div \sqrt{a^2b}$ (ii) $\sqrt[4]{\sqrt[3]{a^2}}$

(iii) $\sqrt{a^{-1}b}.\sqrt{b^{-1}c}\sqrt{c^{-1}a}$

Q.29 If $a^x = b$, $b^y = c$ and $c^z = a$, prove that $xyz = 1$.

Q.30 If $a^x = b^y = c^z$ and $b^2 = ac$, prove that

$y = \frac{2xz}{x+z}$.

Q.31 Assuming that x is a positive real number and a, b, c are rational numbers, show that :

(i) $\left(\frac{x^b}{x^c}\right)^a \left(\frac{x^c}{x^a}\right)^b \left(\frac{x^a}{x^b}\right)^c = 1$

(ii) $\left(\frac{x^a}{x^b}\right)^{1/ab} \left(\frac{x^b}{x^c}\right)^{1/bc} \left(\frac{x^c}{x^a}\right)^{1/ac} = 1$

(iii) $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$

(iv) $\left(\frac{x^a}{x^b}\right)^{a+b} \left(\frac{x^b}{x^c}\right)^{b+c} \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

Q.32 If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$,

prove that m −n = 1.

Q.33 Assuming that x is a positive real number and a, b, c are rational numbers, show that:

(i) $\left(\frac{x^a}{x^b}\right)^{a+b-c} \left(\frac{x^b}{x^c}\right)^{b+c-a} \left(\frac{x^c}{x^a}\right)^{c+a-b} = 1$

(ii) $\left(\frac{x^a}{x^{-b}}\right)^{a^2+b^2-ab} . \left(\frac{x^b}{x^{-c}}\right)^{b^2+c^2-bc} . \left(\frac{x^c}{x^{-a}}\right)^{c^2+a^2-ca} = x^{2(a^3+b^3+c^3)}$

Q.34 If $25^{x-1} = 5^{2x-1} - 100$, find the value of x.

Q.35 Simplify :-

(i) $5\sqrt{2}+20\sqrt{2}$ (ii) $6\sqrt{3}-4\sqrt{3}+9\sqrt{3}$

(iii) $2\sqrt{3}+\sqrt{27}$ (iv) $4\sqrt{3}-3\sqrt{12}+2\sqrt{75}$

Q.36 Simplify : $15\sqrt{6}-\sqrt{216}+\sqrt{96}$

Q.37 Simplify :- (i) $5\sqrt{147}-\frac{4}{3}\sqrt{\frac{1}{3}}+7\sqrt{\frac{1}{3}}$

(ii) $\sqrt{294}-\sqrt{150}+2\sqrt{6}-3\sqrt{\frac{1}{6}}$

Q.38 Simplify by combining similar terms :-

(i) $2.\sqrt[3]{40}+3.\sqrt[3]{625}-4.\sqrt[3]{320}$

(ii) $\sqrt[4]{81}-8.\sqrt[3]{216}+15.\sqrt[5]{32}+\sqrt{225}$

Q.39 Given that $\sqrt{3}$ = 1.7321, find correct to 3 places of decimals, the value of $\sqrt{192}-\frac{1}{2}\sqrt{48}-\sqrt{75}$

Q.40 Multiply :

(i) $3\sqrt{5}$ by $5\sqrt{5}$

(ii) $5\sqrt{2}, 3\sqrt{10}$ and $2\sqrt{15}$

Q.41 Multiply : $\sqrt[3]{4}$ by $\sqrt[3]{22}$

Q.42 Multiply : $\sqrt{14}$ by $\sqrt{21}$

Q.43 Simplify each of the following expressions :-

(i) $(3+\sqrt{3})(2+\sqrt{2})$ (ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$ (iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Q.44 Multiply : $\sqrt[3]{7}$ by $\sqrt{2}$

Q.45 Divide :- $\sqrt[3]{24}$ by $\sqrt[3]{100}$

Q.46 Simplify :- $\frac{\sqrt{a^2-b^2}+a}{\sqrt{a^2+b^2}+b} \div \frac{\sqrt{a^2+b^2}-b}{a-\sqrt{a^2-b^2}}$

Q.47 Simplify and express the result in its simple form :-

(i) $5.\sqrt[3]{4} \div (3\sqrt{2}.\sqrt[3]{3})$

(ii) $9.\sqrt[3]{4} \div (3.\sqrt[3]{2}.\sqrt{3})$

Q.48 Find the rationalizing factors of following :

(i) $\sqrt{10}$ (ii) $\sqrt{162}$ (iii) $\sqrt[3]{4}$

(iv) $\sqrt[3]{16}$ (v) $\sqrt[4]{162}$ (vi) $\sqrt[3]{40}$

Q.49 Find the rationalising factor of : $(\sqrt{3}+\sqrt{10}-\sqrt{5})$

Q.50 Find the simplest rationalising factor of : $2+\sqrt{3}+\sqrt{5}$

Q.51 Rationalise the denominator in each of the following :

(i) $\dfrac{2\sqrt{7}}{\sqrt{11}}$ (ii) $\dfrac{3\sqrt[3]{5}}{\sqrt[3]{9}}$

Q.52 Find the value to three places of decimals; of each of the following. It is given that $\sqrt{2}=1.414$, $\sqrt{3}=1.732$ and $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$ (approx).

(i) $\dfrac{\sqrt{2}+1}{\sqrt{5}}$ (ii) $\dfrac{2-\sqrt{3}}{\sqrt{3}}$ (iii) $\dfrac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$

Q.53 Rationalise :

(i) $\dfrac{1}{\sqrt{7}-\sqrt{6}}$ (ii) $\dfrac{1}{\sqrt{5}+\sqrt{2}}$

Q.54 Simplify each of the following by rationalising the denominator :

(i) $\dfrac{5+\sqrt{6}}{5-\sqrt{6}}$ (ii) $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

Q.55 Simplify the following :

$$\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$$

Q.56 If $\dfrac{3+2\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$, where a and b are rationals. Find the values of a and b

Q.57 If $x=\dfrac{1}{2+\sqrt{3}}$, find the value of $x^3-x^2-11x+3$

Q.58 If $x=3-2\sqrt{2}$, find $x^2+\dfrac{1}{x^2}$

Q.59 If $x=1-\sqrt{2}$, find the value of $\left(x-\dfrac{1}{x}\right)^3$

Q.60 If $x=\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y=\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find x^2+y^2.

Q.61 If $x=1+\sqrt{2}+\sqrt{3}$, prove that $x^4-4x^3-4x^2+16x-8=0$

Q.62 Express the following surd with a rational denominator : $\dfrac{8}{\sqrt{15}+1-\sqrt{5}-\sqrt{3}}$

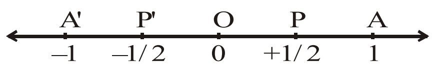
ANSWER KEY

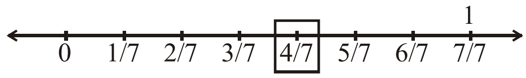
1. $2,\left[\dfrac{7}{2},\dfrac{11}{4},\dfrac{17}{4}\right]5$ **2.** $4,\left[\dfrac{21}{5},\dfrac{22}{5},\dfrac{23}{5},\dfrac{24}{5}\right],5$

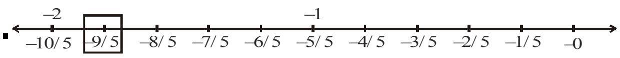
3. $\dfrac{6}{5},\left[\dfrac{5}{4},\dfrac{13}{10},\dfrac{27}{20}\right];\dfrac{7}{5}$ **4.** $\dfrac{7}{8}=0.875$

5. $\dfrac{35}{16}=2.1875$ **6.** $\dfrac{8}{3}=2.6666...=2.\overline{6}$

7. $\dfrac{2}{11}=0.181818\ ....=0.\overline{18}$

8. 

9. 

10. 

11. (i) $\dfrac{3}{20}$ (ii) $\dfrac{27}{40}$ (iii) $\dfrac{-411}{16}$

12. (i) $\dfrac{2}{3}$ (ii) $\dfrac{35}{99}$ (iii) $\dfrac{65}{111}$

13. (i) $\dfrac{47}{9}$ (ii) $\dfrac{2320}{99}$

14 $\dfrac{2}{7}=2\times\dfrac{1}{7}=0.\overline{285714}$; $\dfrac{3}{7}=3\times\dfrac{1}{7}=0.\overline{428571}$; $\dfrac{4}{7}=4\times\dfrac{1}{7}=0.\overline{571428}$; $\dfrac{5}{7}=5\times\dfrac{1}{7}=0.\overline{714285}$

15. (i) $\dfrac{29}{90}$ (ii) $\dfrac{37}{300}$

16. Rational number = 2.5, Irr. no. = $\sqrt{ab}=\sqrt{2\times 3}=\sqrt{6}$

17. $\sqrt{5}$ and $\sqrt{2\times\sqrt{5}}$

18. 1.414213562......... & 1.732050808.........

19. 0.1201001000100001...,0.12101001000100001...

20. 0.25 and 0.2525

21. 0.101, 0.1002000100001......

22. 0.111101001000100001.....

23. (i) irrational. (ii) rational.

24. (i) $\sqrt{5}$ is the square root of a nonperfect square natural number.

$\therefore$ $\sqrt{5}$ is irrational and negative of an irrational number is irrational.

$\therefore$ $-\sqrt{5}$ is irrational.

(ii) We know that the sum of a rational number and an irrational number is always an irrational number

$\therefore$ $(2+\sqrt{6})$ is irrational

[$\because$ 2 is rational and $\sqrt{6}$ is irrational]

(iii) We know that the product of a nonzero rational number and an irrational number is always irrational.

$\therefore$ $5\sqrt{3}$ is irrational.

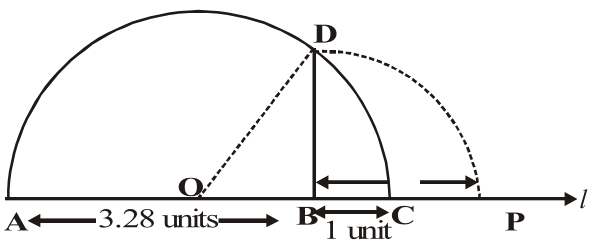
[$\because$ 5 is rational and $\sqrt{3}$ is irrational]

(iv) $(\sqrt{7}-2)$ = [(−2) + $\sqrt{7}$] being the sum of a rational number and an irrational number, is irrational.

(v) $\frac{7}{3\sqrt{5}}=\frac{7}{3\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{7}{15}\sqrt{5}$ which is irrational, being the product of a non-zero rational number and an irrational number.

(vi) $(3+\sqrt{3})^2$ = (9 + 3 + $6\sqrt{3}$) = (12 + $6\sqrt{3}$) which is irrational, being the sum of a rational number and an irrational number.

25.



26. (i) 800 (ii) 64 (iii) $\frac{343}{729}$ (iv) $\frac{125}{8}$ (v) $\frac{16}{25}$

27. (i) $\frac{1}{36}$ (ii) $\frac{2197}{1331}$ (iii) $\frac{1}{(27)^{\frac{1}{2}}}$ (iv) $\frac{1}{2}$ (v) $\frac{1}{(5)^{\frac{19}{2}}}$

(vi) 175

28. (i) $\left(\frac{b}{a^4}\right)^{\frac{1}{6}}$ (ii) $a^{\frac{1}{6}}$ (iii) 1

34. 2

35. (i) $25\sqrt{2}$ (ii) $11\sqrt{3}$ (iii) $5\sqrt{3}$ (iv) $8\sqrt{3}$

36. $13\sqrt{6}$

37. (i) $\frac{332}{9}\sqrt{3}$ (ii) $\frac{7}{2}\sqrt{6}$

38. (i) $3.\sqrt[3]{5}$ (ii) 0

39. 1.732

40. (i) 75 (ii) 300 $\sqrt{3}$

41. $2\sqrt[3]{11}$

42. $7\sqrt{6}$

43. (i) $6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$ (ii) 6 (iii)7 + $2\sqrt{10}$ (iv)3

44. $\sqrt[6]{392}$

45. $\sqrt[3]{\frac{6}{25}}$

46. $\frac{b^2}{a^2}$

47. (i) $\frac{5}{3}\times\sqrt[6]{\frac{2}{9}}$ (ii) $3\,.\sqrt[6]{\frac{4}{27}}$

48. (i) $\sqrt{10}$ (ii) $\sqrt{2}$ (iii) $\sqrt[3]{4^2}$ (iv) $\sqrt[3]{2^2}$ (v) $\sqrt[4]{8}$ (vi) $5^{\frac{2}{3}}$

49. $(3+\sqrt{10}+\sqrt{5})(8-2\sqrt{30})$

50. $(2+\sqrt{3}-\sqrt{5})(1-2\sqrt{3})$

51. (i) $\frac{2}{11}\sqrt{77}$ (ii) $\sqrt[3]{15}$

52. (i)1.079 (ii) 0.154 (iii)0.654

53. (i) $\sqrt{7}+\sqrt{6}$ (ii) $\frac{1}{3}(\sqrt{5}-\sqrt{2})$

54. (i) $\frac{31+10\sqrt{6}}{19}$ (ii) $6-\sqrt{35}$

55. 0

56. a = $\frac{13}{7}$, b = $\frac{9}{7}$

57. 0

58. 34

59. 8

60. 98

62. $(\sqrt{15}+1+\sqrt{5}+\sqrt{3})$

EXERCISE – II | SCHOOL EXAM/BOARD

Q.1 Express the following in the form of p/q.
(i) $.\overline{3}$ (ii) $.\overline{37}$

Q.2 Write two irrational numbers between 0.2 and 0.21.

Q.3 Write three irrational numbers between 0.20200200020000 2 . . . and 0.203003000300003...

Q.4 Write three irrational numbers between $\sqrt{3}$ and $\sqrt{5}$

Q.5 Find two irrational numbers between 0.5 and 0.55.

Q.6 Find two irrational numbers lying between 0.1 and 0.12.

Q.7 Given a rational approximation of $\sqrt{3}$ correct to two places of decimals.

Q.8 Express 2 as a surd of fifth order.

Q.9 Express $\sqrt[3]{2}$ as a surd of order 12.

Q.10 Express $\sqrt[24]{49}$ as a surd of order 12.

Q.11 In the following express the result in the simplest form : $\sqrt[3]{-108a^4b^3}$

Q.12 Express as a pure surd : $\frac{1}{3}\sqrt[3]{54}$

Q.13 Simplify : $2.\sqrt[3]{40} + 3.\sqrt[3]{625} + 4.\sqrt[3]{320}$

Q.14 Simplify : $(3\sqrt{5}-2\sqrt{3})(3\sqrt{5}+2\sqrt{3})$

Q.15 Simplify : $\sqrt{m^2n^2} \times \sqrt[6]{m^2n^2} \times \sqrt[3]{m^2n^2}$

Q.16 Simplify : $\sqrt[5]{\sqrt[4]{(2^4)^3}} - 5\sqrt[5]{8} + 2\sqrt[4]{\sqrt[5]{(2^3)^4}}$

Q.17 If $\sqrt{3} = 1.732$, find the value of $\frac{2}{\sqrt{3}}$.

Q.18 Which of the following is
(i) rational (ii) irrational number
(A) $(2+\sqrt{3})^2$ (B) $(3+\sqrt{4})^2$

Q.19 Which of the following numbers are
(i) rational (ii) irrational
(A) $(5+\sqrt{3})^2$ (B) $(2+\sqrt{3})\,(2-\sqrt{3})$

Q.20 Given that $\sqrt{3} = 1.732$, find the value of
$$\sqrt{75}+\frac{1}{2}\sqrt{48} - \sqrt{192}$$

Q.21 Determine a and b if
$$\frac{5+\sqrt{3}}{7-4\sqrt{3}} = 94a + 3\sqrt{3}\,b$$

Q.22 If $\sqrt{5} = 2.236$ and $\sqrt{6} = 2.449$, find the value of $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Q.23 If $x = 7+4\sqrt{3}$, find the value of $\sqrt{x}+\frac{1}{\sqrt{x}}$

Q.24 If $p = 3-2\sqrt{2}$, determine $p^2 + \frac{1}{p^2}$

Q.25 Find the simplest rationalising factor of $\sqrt{5}+\sqrt{3}+2$

Q.26 Express $\sqrt[4]{3}$, $\sqrt[6]{4}$, $\sqrt[3]{2}$ and $\sqrt[24]{81}$ as surds of order 12.

Q.27 Simplify : $3\sqrt{2} + \sqrt[4]{64} + \sqrt[4]{2500} + \sqrt[6]{8}$

Q.28 Simplify and express the results in simplest form : $\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y} \div \frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}$

Q.29 Simplify by rationalising the denominator :
$$\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}.$$

Q.30 Find x if $x = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

Q.31 Express with a rational denominator :
$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

Q.32 Express with a rational denominator :
$$\frac{1}{\sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}}$$

Q.33 Find x if $x = \frac{2\left(\sqrt{2}+\sqrt{6}\right)}{3\sqrt{2+\sqrt{3}}}$

Q.34 Evaluate : $\sqrt{5+2\sqrt{6}}$

Q.35 If $a = 1-\sqrt{2}$, find the value of $\left(a-\frac{1}{a}\right)^3$.

Q.36 If $x = \frac{\sqrt{3}+1}{2}$, find the value of $4x^3 + 2x^2 - 8x + 7$.

Q.37 If $x = 6 - \sqrt{35}$, find $x^2 + \frac{1}{x^2}$

Q.38 If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find the value of $x^2 + y^2 + xy$.

Q.39 If $x = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $y = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find the value of $x^2 - y^2$.

Q.40 Given $\sqrt{2} = 1.4142$, $\sqrt{3} = 1.7321$ and $\sqrt{5} = 2.236$, find correct to three places of decimals the value of

$$\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$$

Q.41 Determine rational numbers p and q if $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = p - 7\sqrt{5}q$

Q.42 Taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{6} = 2.449$, find the value of the following : $\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

Q.43 Simplify : $\frac{6}{2\sqrt{3}-\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

Q.44 Simplify : $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} + \frac{2\sqrt{3}}{\sqrt{6}+2} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

Q.45 Show that $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}}$

$$-\frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$$

Q.46 Determine rational numbers a and b if $\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + 3\sqrt{3}b$

Q.47 $x = 3 + 2\sqrt{2}$, find the value of $x^4 + \frac{1}{x^4}$

Q.48 Simplify $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

Q.49 If $x = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $y = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, find the value of $3x^2 + 4xy - 3y^2$

ANSWER KEY

1. (i) $\frac{1}{3}$ (ii) $\frac{37}{99}$
2. 0.2010010001........., 0.2020020002..........
3. 0.20201001000100001..........,
0.202020020002..., 0.202030030003..........
4. 1.8010010001......., 1.9010010001.......,
2.010010001.......
5. 0.501001001....... and 0.5020020002.......
6. 0.10100100010000....... and
0.1020020002.......
7. 1.73 **8.** $\sqrt[5]{32}$ **9.** $\sqrt[12]{16}$
10. $\sqrt[12]{7}$ **11.** $-3ab\sqrt[3]{4a}$ **12.** $\sqrt[3]{2}$
13. $35\sqrt[3]{5}$ **14.** 33 **15.** m^2n^2
16. $-2.\sqrt[5]{8}$ **17.** 1.154
18. (a) irrational (b) rational
19. (a) irrational (b) rational
20. – 1.732 **21.** $a = \frac{1}{2}$, $b = 9$
22. – 0.213 **23.** 4 **24.** 34
25. $(2+\sqrt{3}-\sqrt{5})$ $(1 - 2\sqrt{3})$
26. $\sqrt[12]{27}, \sqrt[12]{16}, \sqrt[12]{16}, \sqrt[12]{9}$ **27.** $11\sqrt{2}$ **28.** $\frac{y^2}{x^2}$
29. $\frac{114-41\sqrt{6}}{30}$ **30.** $\sqrt{2}$ **31.** $\sqrt{10}+\sqrt{5}$
32. $\frac{\sqrt{21}+\sqrt{10}-\sqrt{14}-\sqrt{15}}{2}$ **33.** $\frac{4}{3}$ **34.** $\sqrt{3}+\sqrt{2}$
35. 8 **36.** 7 **37.** 142 **38.** 99
39. $-144\sqrt{5}$ **40.** 2.063 **41.** $p = 0$, $q = \frac{-1}{11}$
42. 14.268 **43.** 0 **44.** 0
46. a = 4, b = 0 **47.** 1154
48. 1 **49.** $\frac{12+56\sqrt{10}}{3}$

EXERCISE – III OLYMPIAD QUESTIONS

Q.1 If x, y, z be rational numbers such that $x > y$ and $z < y$ then

(A) $z > x$ (B) $z < x$

(C) $y < z$ (D) $y < x$

Q.2 For any two rational numbers x and y, which of the following properties are correct ?

(i) $x < y$ (ii) $x = y$ (iii) $x > y$

(A) Only (i) and (ii) are correct

(B) Only (ii) and (iii) are correct

(C) Only (ii) is correct

(D) All (i), (ii) and (iii) are correct

Q.3 The number $\frac{3-\sqrt{3}}{3+\sqrt{3}}$ is

(A) rational (B) irrational

(C) both (D) can't say

Q.4 The rational number between $\frac{1}{2}$ and $\frac{1}{3}$ is

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

Q.5 If A : The quotient of two integers is always a rational number and R : $\frac{1}{0}$ is not rational, then which of the following statements is true ?

(A) A is true and R is the correct explanation of A

(B) A is false and R is the correct explanation of A

(C) A is true and R is false

(D) Both A and R are false

Q.6 The two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$ are

(A) $2^{\frac{1}{2}}, 6^{\frac{1}{4}}$ (B) $3^{\frac{1}{4}}, 3^{\frac{1}{6}}$

(C) $6^{\frac{1}{8}}, 3^{\frac{1}{4}}$ (D) none

Q.7 The number $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$ where $x, y > 0$ is

(A) rational (B) irrational

(C) both (D) none

Q.8 The sum of rational and irrational number is always

(A) rational (B) irrational

(C) both (D) can't say

Q.9 The product of rational and irrational number is always

(A) rational (B) irrational

(C) both (D) can't say

Q.10 The number $(6+\sqrt{2})(6-\sqrt{2})$ is

(A) rational (B) irrational

(C) can't say (D) none

Q.11 Which of the following numbers has the terminal decimal representation?

(A) $\frac{1}{7}$ (B) $\frac{1}{3}$

(C) $\frac{3}{5}$ (D) $\frac{17}{3}$

Q.12 The ascending order of the following surds $\sqrt[9]{2}, \sqrt[6]{3}, \sqrt[9]{4}$ is

(A) $\sqrt[9]{4}, \sqrt[6]{3}, \sqrt[3]{2}$ (B) $\sqrt[9]{4}, \sqrt[3]{2}, \sqrt[6]{3}$

(C) $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$ (D) $\sqrt[6]{3}, \sqrt[9]{4}, \sqrt[3]{2}$

Q.13 Which of the following is a pure surd ?

(A) $4\sqrt{3}$ (B) $3\sqrt[3]{5}$

(C) $\sqrt{12}$ (D) $\frac{3}{4}\sqrt{8}$

Q.14 The greatest among $\sqrt[3]{4}, \sqrt[4]{5}, \sqrt[4]{3}$ is

(A) $\sqrt[3]{4}$ (B) $\sqrt[4]{5}$

(C) $\sqrt[4]{3}$ (D) none of these

Q.15 The greater among $\sqrt{17}-\sqrt{12}$ and $\sqrt{11}-\sqrt{6}$ is

(A) $\sqrt{17}-\sqrt{12}$ (B) $\sqrt{11}-\sqrt{6}$

(C) both are equal (D) can't say

Q.16 Which of the following is a rational number

(A) $\sqrt{5}$ (B) $\sqrt{6}$

(C) $\sqrt{8}$ (D) $\sqrt{9}$

Q.17 Representation of $3.\overline{6}$ in rational

(A) $\frac{11}{3}$ (B) $\frac{3}{11}$

(C) $\frac{36}{10}$ (D) $\frac{33}{10}$

Q.18 The value of b if $f(x) = x^2 + 4\sqrt{x} + b$ and $f(16) = 275$ is
(A) 3 (B) 2
(C) 1 (D) 0

Q.19 The value of a and b if $f(x) = ax + b$ and $f(2) = 8$, $f(3) = 11$ is
(A) $a = 3, b = -2$ (B) $a = -3, b = 2$
(C) $a = -3, b = -2$ (D) $a = 3, b = 2$

Q.20 The distance between -3 and $|-3|$ is
(A) 6 (B) 0
(C) can't say (D) none

Q.21 The given rational numbers are $\frac{1}{2}, \frac{4}{-5}, \frac{-7}{8}$. If these numbers are arranged in the ascending order or descending order, then the middle number is
(A) $\frac{1}{2}$ (B) $\frac{-7}{8}$
(C) $\frac{4}{-5}$ (D) None

Q.22 The value of x in $|x - 2| = 12$ is
(A) 14, 10 (B) 14, −10
(C) −14, −10 (D) −14, 10

Q.23 Solution of $|2x - 1| \geq 5$ is
(A) $x \geq -2, x \geq 3$ (B) $x \leq -2, x \leq 3$
(C) $x \leq -2, x \geq 3$ (D) $x \leq -2, x \leq 3$

Q.24 The number $(\sqrt{2}+\sqrt{3})^2$ is
(A) rational number (B) irrational number
(C) can't say (D) none

Q.25 The average of the middle two rational numbers if $\frac{4}{7}, \frac{1}{3}, \frac{2}{5}, \frac{5}{9}$ are arranged in ascending order is
(A) $\frac{86}{90}$ (B) $\frac{86}{45}$
(C) $\frac{43}{45}$ (D) $\frac{43}{90}$

Q.26 What is the percentage of least number in the greatest number if $\frac{3}{5}, \frac{9}{5}, \frac{1}{5}, \frac{7}{5}$ are arranged in ascending or desending order ?
(A) $11\frac{1}{9}\%$ (B) 10%
(C) 20% (D) 25%

Q.27 The irrational number between 2 and 3 is
(A) $\sqrt{2}$ (B) $\sqrt{3}$
(C) $\sqrt{5}$ (D) $\sqrt{11}$

Q.28 The value of a if $f(x) = \frac{1}{x} + ax$ and $f\left(\frac{1}{5}\right) = \frac{28}{5}$
(A) 3 (B) 2
(C) 1 (D) 0

Q.29 $\frac{217}{143}$ can be expressed decimal from as
(A) $1.51\overline{7}$ (B) $1.\overline{517}$
(C) $1.5\overline{17}$ (D) 1.517...

Q.30 The equivalent rational form of $17.\overline{6}$ is
(A) $\frac{53}{3}$ (B) $\frac{88}{5}$
(C) $\frac{44}{25}$ (D) none

Q.31 The value of x if $|3x + 2| = 8$
(A) 2 (B) −2
(C) $\frac{10}{3}, -2$ (D) $-\frac{10}{3}, 2$

Q.32 $\frac{961}{625}$ is
(A) terminating decimal
(B) nonterminating decimal
(C) cannot be determined
(D) none of these

Q.33 2.003 can be expressed in the rational form as
(A) $\frac{2003}{100}$ (B) $\frac{2003}{1000}$
(C) $\frac{2003}{10000}$ (D) $\frac{2003}{10}$

Q.34 Rational number between $\sqrt{2}$ and $\sqrt{3}$ is
(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}\times\sqrt{3}}{2}$
(C) 1.5 (D) 1.8

Q.35 Which of the following is not a rational number?
(A) $\sqrt{2}$ (B) $\sqrt{4}$
(C) $\sqrt{9}$ (D) $\sqrt{16}$

Q.36 Set of natural numbers is a subset of
(A) set of even number
(B) set of odd numbers
(C) set of composite numbers
(D) set of real numbers

Q.37 Which of the following statement is false ?

(A) Every fraction is a rational number

(B) Every rational number is a fraction

(C) Every integer is a rational number

(D) All the above

Q.38 A rational number can be expressed as a terminating decimal if the denominator has factors

(A) 2 or 5 (B) 2, 3 or 5

(C) 3 or 5 (D) none of these

Q.39 Express 0.75 as rational number.

(A) $\frac{75}{99}$ (B) $\frac{75}{90}$

(C) $\frac{3}{4}$ (D) None

Q.40 $\sqrt{a} > \sqrt{b} > \sqrt{c} > \sqrt{d}$ where d, c, b a are consecutive natural numbers. Then which of the following is true ?

(A) $\sqrt{a}-\sqrt{b} > \sqrt{c}-\sqrt{d}$ (B) $\sqrt{c}-\sqrt{d} > \sqrt{a}-\sqrt{b}$

(C) $\sqrt{a}-\sqrt{c} > \sqrt{b}-\sqrt{d}$ (D) None of these

Q.41 The smaller among the following surds is

$\sqrt{\frac{1}{2}}, \sqrt[3]{\frac{2}{3}}, \sqrt{\frac{1}{3}}, \sqrt{\frac{1}{4}}$

(A) $\sqrt{\frac{1}{2}}$ (B) $\sqrt[3]{\frac{2}{3}}$

(C) $\sqrt{\frac{1}{3}}$ (D) $\sqrt{\frac{1}{4}}$

Q.42 The product of $\sqrt[3]{2}, \sqrt[4]{3}$ is

(A) $(234)^{\frac{1}{12}}$ (B) $(324)^{\frac{1}{12}}$

(C) $(432)^{\frac{1}{12}}$ (D) $(433)^{\frac{1}{12}}$

Q.43 Divide $\sqrt[6]{12}$ by $\sqrt{3}\ \sqrt[3]{2}$.

(A) $\frac{1}{\sqrt[2]{3}}$ (B) $\frac{1}{\sqrt[3]{3}}$

(C) $\frac{1}{\sqrt[4]{3}}$ (D) $\frac{1}{\sqrt[5]{3}}$

Q.44 The rationalising factor of $2\sqrt[3]{5}$ is

(A) $\sqrt[3]{5}$ (B) $\sqrt[3]{5^2}$

(C) 5^2 (D) 5^3

Q.45 The rationalising factor of $\sqrt[5]{a^2b^3c^4}$ is

(A) $\sqrt[5]{a^3b^2c}$ (B) $\sqrt[4]{a^3b^2c}$

(C) $\sqrt[3]{a^3b^2c}$ (D) $\sqrt{a^3b^2c}$

Q.46 The rationalising factor of $\sqrt{108}$ is

(A) $\sqrt{3}$ (B) $\sqrt[3]{3}$

(C) $\sqrt[3]{27}$ (D) $\sqrt[3]{15}$

Q.47 The rational denominator of the surd $\frac{3\sqrt[3]{5}}{\sqrt[3]{9}}$ is

(A) 1 (B) 2

(C) 3 (D) 4

Q.48 Given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$.

Then the value of $\frac{1}{\sqrt{10}}$ up to three decimal places is

(A)2.414 (B) 0.316

(C)1.079 (D) 3.162

Q.49 $\frac{-3}{0}$ is ....

(A) positive rational number

(B) negative rational number

(C) either positive or negative rational number

(D) neither positive nor negative rational number

Q.50 A rational number equivalent to $\frac{-5}{-3}$ is

(A) $\frac{-25}{15}$ (B) $\frac{25}{-15}$

(C) $\frac{25}{15}$ (D) none of these

Q.51 $\frac{-2}{-19}$ is a

(A) positive rational number

(B) negative rational number

(C) either positive or negative rational number

(D) neither positive nor negative rational number

Q.52 The rational number $\frac{0}{7}$

(A) has a positive numerator

(B) has negative numerator

(C) has either a positive numerator or a negative numerator

(D) has neither a positive numerator nor a negative numerator

Q.53 Which of the following rational numbers is in the standard form ?

(A) $\frac{8}{-36}$ (B) $\frac{-7}{56}$

(C) $\frac{3}{-4}$ (D) None

Q.54 Which of the following statement is true ?

(A) $\frac{3}{-8} > \frac{-12}{32}$ (B) $\frac{3}{-8} - \frac{-12}{32}$

(C) $\frac{3}{-8} < \frac{-12}{32}$ (D) $\frac{3}{5} > \frac{4}{3}$

Q.55 If $\frac{-3}{5} = \frac{-24}{x}$, then x is

(A) 40 (B) – 40

(C) ± 40 (D) none

Q.56 If $\frac{-3}{x} = \frac{x}{27}$ then x is

(A) a rational number

(B) not a rational number

(C) an integer

(D) a natural number

Q.57 A rational number $\frac{-2}{3}$

(A) lies to the left side of 0 on the number line

(B) lies to the right side of 0 on the number line

(C) it is not possible to represent on the number line

(D) cannot be determined on which side the number lies

Q.58 Which of the following statement is true?

(A) $\frac{-5}{8}$ lies to the left of 0 on the number line

(B) $\frac{3}{7}$ lies to the right at 0 on the number line

(C) The rational numbers $\frac{1}{3}$ and $\frac{-7}{3}$ are on opposite sides of 0 on the number line

(D) All the above

Q.59 Out of the rational numbers $\frac{-5}{11}, \frac{5}{-12}, \frac{-5}{17}$, which is greater ?

(A) $\frac{-5}{11}$ (B) $\frac{5}{-12}$

(C) $\frac{-5}{17}$ (D) None

Q.60 Out of the rational numbers $\frac{7}{-13}, \frac{-5}{13}, \frac{-11}{13}$ which is smaller ?

(A) $\frac{7}{-13}$ (B) $\frac{-5}{13}$

(C) $\frac{-11}{13}$ (D) None

Q.61 If both 'a' and 'b' are rational numbers then 'a' and 'b' from the following $\frac{3-\sqrt{5}}{3+2\sqrt{5}} - a\sqrt{5} - b$ are

(A) $a = \frac{9}{11}, b = \frac{19}{11}$ (B) $a = \frac{19}{11}, b = \frac{9}{11}$

(C) $a = \frac{2}{11}, b = \frac{-8}{11}$ (D) $a = \frac{10}{11}, b = \frac{21}{11}$

Q.62 The value of $\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$ is

(A) $-\sqrt{5}$ (B) $-2\sqrt{5}$

(C) $-4\sqrt{5}$ (D) $-8\sqrt{5}$

Q.63 If $x = 2 - \sqrt{3}$ then the value of $x^2 + \frac{1}{x^2}$ and $x^2 - \frac{1}{x^2}$ is

(A) $14, 8\sqrt{3}$ (B) $-14, -8\sqrt{3}$

(C) $14, -8\sqrt{3}$ (D) $-14, 8\sqrt{3}$

Q.64 The value of $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}}$

(A) 0 (B) 1

(C) 2 (D) 4

Q.65 If $x = 3 + \sqrt{8}$ then $x^3 + \frac{1}{x^3} =$

(A) 216 (B) 198

(C) 192 (D) 261

Q.66 If $x = \frac{\sqrt{3}+1}{2}$ then the value of $4x^3 + 2x^2 - 8x + 7$ is

(A) 10 (B) 8

(C) 6 (D) 4

Q.67 If $x = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$, $y = \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ then the value of $x^2 + xy + y^2$ is

(A) $\frac{4(a-b)}{(a+b)}$ (B) $\frac{4(a+b)}{(a-b)}$

(C) $\frac{2(a+b)}{(a-b)}$ (D) $\frac{2(a-b)}{(a+b)}$

Q.68 The smallest positive number from the numbers below is

(A) $10 - 3\sqrt{11}$ (B) $3\sqrt{11} - 10$

(C) $18 - 5\sqrt{13}$ (D) $51 - 10\sqrt{26}$

Q.69 $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ equals

(A) $\sqrt{2}+\sqrt{3}-\sqrt{5}$ (B) $4-\sqrt{2}-\sqrt{3}$

(C) $\sqrt{2}+\sqrt{3}+\sqrt{6}-5$ (D) $\frac{1}{2}(\sqrt{2}+\sqrt{5}-\sqrt{3})$

Q.70 The value of $\left(\sqrt[6]{27}-\sqrt{6\frac{3}{4}}\right)^2$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{2}$

(C) $\frac{\sqrt{3}}{4}$ (D) $\frac{3}{4}$

Q.71 Which of the following is closest to $\sqrt{65}-\sqrt{63}$?

(A) 0.12 (B) 0.25

(C) 0.14 (D) 0.15

Q.72 The value of $\sqrt{8}+\sqrt{18}$ is

(A) $\sqrt{26}$ (B) $2(\sqrt{2}+\sqrt{3})$

(C) 7 (D) $5\sqrt{2}$

Q.73 The fraction $\frac{2(\sqrt{2}+\sqrt{6})}{2\left(\sqrt{2+\sqrt{3}}\right)}$ is equal to

(A) $\frac{2\sqrt{2}}{3}$ (B) 1

(C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{4}{3}$

Q.74 If $N = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}}$ then N equals to

(A) 1 (B) $2\sqrt{2}-1$

(C) $\frac{\sqrt{5}}{2}$ (D) None of these

Q.75 If $t = \frac{1}{1-\sqrt[4]{2}}$ then t equal to

(A) $(1-\sqrt[4]{2})(2-\sqrt{2})$ (B) $(1-\sqrt[4]{2})(1+\sqrt{2})$

(C) $-(1+\sqrt[4]{2})(1+\sqrt{2})$ (D) $(1+\sqrt[4]{2})(1+\sqrt{2})$

Q.76 If $x = \sqrt{3}+\sqrt{2}$ then $x^2 + \frac{1}{x^2}$ is

(A) $2\sqrt{3}$ (B) 10

(C) 12 (D) 14

Q.77 The biggest surd among $\sqrt[3]{2}$; $\sqrt{3}$, $\sqrt[3]{5}$ is

(A) $\sqrt[3]{2}$ (B) $\sqrt{3}$

(C) $\sqrt[3]{5}$ (D) None

Q.78 The value of the surd $4\sqrt{3}-3\sqrt{12}+2\sqrt{75}$ is

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$

(C) $6\sqrt{3}$ (D) $8\sqrt{3}$

Q.79 The product of $\sqrt[3]{4}$ and $\sqrt[3]{22}$ is

(A) $2\sqrt[3]{11}$ (B) $3\sqrt[3]{11}$

(C) $4\sqrt[3]{11}$ (D) none

Q.80 The value of $\frac{a+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}+b} + \frac{\sqrt{a^2+b^2}-b}{a-\sqrt{a^2-b^2}}$

(A) $\frac{a^2}{b^2}$ (B) $\frac{b^2}{a^2}$

(C) $\frac{a}{b}$ (D) None

Q.81 If p : Every fraction is a rational number and q : Every rational number is a fraction, then which of the following is correct ?

(A) p is true and q is false

(B) p is false and q is true

(C) Both p and q are true

(D) Both p and q are false

Q.82 Which of the following is a rational number(s)?

(A) $\frac{-2}{9}$ (B) $\frac{4}{-7}$

(C) $\frac{-3}{-17}$ (D) All the three

Q.83 If p : All integers are rational numbers and q : Every rational number is an integer, then which of the following statement is correct?

(A) p is true and q is false

(B) p is false and q is true

(C) Both p and q are true

(D) Both p and q are false

Q.84 If A : if the denominator of a rational number has 2 as a prime factor, then that rational number can be expressed as a terminating decimal and R : $\frac{83}{64}$ is a terminating decimal, then which of the following statements is correct ?

(A) A is false and R is true

(B) A is true and R is false

(C) A is true and R is an example of A

(D) A is false and R is an example supporting A

Q.85 If x and y are two rational numbers, then which of the following statements is wrong ?

(A) $|x + y| \le |x| + |y|$

(B) $|x \times y| = |x| \times |y|$

(C) $|x - y| \le |x| - |y|$

(D) None of these

Q.86 Which of the following statements is true ?

(A) $\frac{-2}{3}<\frac{4}{-9}<\frac{-5}{12}<\frac{7}{-18}$ (B) $\frac{7}{-18}<\frac{-5}{12}<\frac{4}{-9}<\frac{-2}{3}$

(C) $\frac{4}{-9}<\frac{7}{-18}<\frac{-5}{12}<\frac{-2}{3}$ (D) $\frac{-5}{12}<\frac{-2}{3}<\frac{4}{-9}<\frac{7}{-18}$

Q.87 The difference between the greatest and least number of $\frac{5}{9}, \frac{1}{9}, \frac{11}{9}$ is

(A) $\frac{2}{9}$ (B) $\frac{4}{9}$

(C) $\frac{10}{9}$ (D) $\frac{2}{3}$

Q.88 $0.\overline{018}$ can be expressed in the rational form as

(A) $\frac{18}{1000}$ (B) $\frac{18}{990}$

(C) $\frac{18}{9900}$ (D) $\frac{18}{999}$

Q.89 $2.53\overline{6}$ can be expressed in the rational form as

(A) $\frac{716}{300}$ (B) $\frac{761}{3000}$

(C) $\frac{761}{300}$ (D) $\frac{761}{3000}$

Q.90 $0.\overline{23}+0.\overline{22}=$

(A) $0.\overline{45}$ (B) $0.\overline{43}$

(C) $0.4\overline{5}$ (D) 0.45

Q.91 Which of the following statement(s) is true

(A) $|x \times y| = |x| \cdot |y|$, where x and y are rational numbers

(B) Infinite number of rational numbers lie between any two rational numbers

(C) $|x| = -x$ if $x < 0$ where x is a rational number

(D) All the above

Q.92 Express $0.3\overline{58}$ as rational number

(A) $\frac{358}{1000}$ (B) $\frac{358}{999}$

(C) $\frac{355}{990}$ (D) All

Q.93 Which of the following statement is true ?

(A) $\frac{5}{7}<\frac{7}{9}<\frac{9}{11}<\frac{11}{13}$ (B) $\frac{11}{13}<\frac{9}{11}<\frac{7}{9}<\frac{5}{7}$

(C) $\frac{5}{7}<\frac{11}{13}<\frac{7}{9}<\frac{9}{11}$ (D) $\frac{5}{7}<\frac{9}{11}<\frac{11}{13}<\frac{7}{9}$

Q.94 A rational number between $\frac{1}{4}$ and $\frac{1}{3}$ is

(A) $\frac{7}{24}$ (B) 0.29

(C) $\frac{13}{48}$ (D) all the above

Q.95 If A : Every whole number is a natural number and R : 0 is not a natural number, then which of the following statement is true?

(A) A is false and R is the correct explanation of A

(B) A is true and R is the correct explanation of A

(C) A is true and R is false

(D) Both A and R are true

Q.96 $2-\frac{11}{39}+\frac{5}{26}=......$

(A) $\frac{149}{39}$ (B) $1+\frac{71}{78}$

(C) $\frac{149}{76}$ (D) $\frac{149}{98}$

Q.97 $\frac{-143}{21}-$ ......

(A) $-6+\frac{17}{21}$ (B) $6+\left(\frac{-17}{21}\right)$

(C) $(-6)+\left(\frac{-17}{21}\right)$ (D) none

Q.98 Addition of rational numbers does not satisfy which of the following property?

(A) Commutative (B) Associative

(C) Closure (D) None

Q.99 $\frac{-7}{5}+\left(\frac{2}{-11}+\frac{-13}{25}\right)=\left(\frac{-7}{5}+\frac{2}{-11}\right)+\frac{-13}{25}$

This property is

(A) closure (B) commutative

(C) associative (D) identity

Q.100 Which of the following statement is correct ?

(A) 0 is called the additive identity for rational numbers.

(B) 1 is called the multiplicative identity for rational numbers.

(C) The additive inverse of 0 is zero itself.

(D) All the above

Q.101 The sum of two rational numbers is –3. If one of the numbers is $\frac{-7}{5}$, then the other number is

(A) $\frac{-8}{5}$ (B) $\frac{8}{5}$

(C) $\frac{-6}{5}$ (D) $\frac{6}{5}$

Q.102 What number should be added to $\frac{-5}{6}$ so as to get $\frac{3}{2}$?

(A) $\frac{-7}{3}$ (B) $2\frac{1}{3}$

(C) $\frac{8}{3}$ (D) $\frac{-8}{3}$

Q.103 Which of the following alternatives is wrong ? Given that

(i) difference of two rational numbers is a rational number

(ii) subtraction is commutative on rational numbers

(ii) addition is not commutative on rational numbers

(A) (ii) and (iii) (B) (i) only

(C) (i) and (iii) (D) All the above

Q.104 Which of the following statements is true

(A) The reciprocals 1 and –1 are themselves

(B) 0 has no reciprocal

(C) The product of two rational numbers is a rational number

(D) All the above

Q.105 Which is the property of multiplication

$-\frac{4}{3}\left(\frac{-6}{5}+\frac{8}{7}\right)=\left(\frac{-4}{3}\times\frac{-6}{5}\right)+\left(\frac{-4}{3}\times\frac{8}{7}\right)$

(A) Associative property

(B) commutative property

(C) distributive property

(D) none of these

Q.106 The product of a rational number and its reciprocal is

(A) 0 (B) 1

(C) –1 (D) none

Q.107 The product of two rational numbers is $-\frac{9}{16}$. If one of the numbers is $\frac{-4}{3}$, the other number is

(A) $\frac{36}{48}$ (B) $\frac{25}{64}$

(C) $\frac{27}{49}$ (D) $\frac{27}{64}$

Q.108 By what rational number should $\frac{-8}{39}$ be multiplied to obtain 26 ?

(A) $\frac{507}{4}$ (B) $\frac{-507}{4}$

(C) $\frac{407}{4}$ (D) None

Q.109 How many pieces of equal size can but cut from a rope of 30 meters long, each measuring $3\frac{3}{4}$ meters ?

(A) 8 (B) 10

(C) 6 (D) 12

Q.110 If A : Rational number are always closed under division and R : Division by zero is not defined, then which of the following statement is correct ?
(A) A is true and R is the correct explanation of A
(B) A is false and R is the correct explanation of A
(C) A is true and R is false
(D) None of these

Q.111 π is
(A) rational (B) irrational
(C) imaginary (D) an integer

Q.112 The set of all irrational numbers is closed for
(A) addition (B) multiplication
(C) division (D) none of these

Q.113 The additive inverse of $\frac{-a}{b}$ is
(A) $\frac{a}{b}$ (B) $\frac{b}{a}$
(C) $\frac{-b}{a}$ (D) $\frac{-a}{b}$

Q.114 Multiplicative inverse of '0' is
(A) $\frac{1}{0}$ (B) 0
(C) does not exist (D) none of these

Q.115 Express $0.\overline{75}$ as rational number.
(A) $\frac{75}{90}$ (B) $\frac{25}{33}$
(C) $\frac{3}{4}$ (D) None

Q.116 An irrational number is
(A)a terminating and nonepreating decimal
(B) a nonterminating and non repeating decimal
(C) a terinating and repeating decimal
(D) a nonterminating and repeating decimal

Q.117 Which of the following statement is true ?
(A) Every point on the number line represents a rational number
(B) Irrational number cannot be represent on the number line
(C) $\frac{22}{7}$ is a rational number
(D) None of these

Q.118 The set of real numbers does not have the property of
(A) multiplicative inverse
(B) additive inverse
(C) multiplicative identity
(D) none of these

Q.119 Which step in the following problem is wrong ?
a = b = 1 a = b
Step-1 = a^2 = ab
Step-2 = $a^2 - b^2 = ab - b^2$
Step-3 = (a + b) (a − b) = b (a − b)
Step-4 : a + b = $\frac{b(a-b)}{a-b}$
a + b = b 1 + 1 = 1 2 = 1
(A) Step-4 (B) Step-3
(C) Step-2 (D) Step-1

Q.120 If 'm' is an irrational number then '2m' is _____ .
(A) a rational number (B) an irrational number
(C) a whole number (D) a natural number

Q.121 The value of $\sqrt{3}$ is
(A) 1.414 (B) 2.256
(C) 1.732 (D) none

Q.122 The greatest among the following is
I. $\sqrt[3]{1.728}$ II. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
III. $\left(\frac{1}{2}\right)^{-2}$ IV. $\frac{17}{8}$
(A) I (B) IV
(C) II (D) III

Q.123 A fraction $\frac{a}{b}$ can be expressed as a terminating decimal, if b has no prime factors other than
(A) 2, 3 (B) 3, 5
(C) 2, 5 (D) 2, 3, 5

Q.124 The sum of a rational and an irrational number is
(A) an irrational number
(B) a rational number
(C) an integer
(D) a whole number

Q.125 The product of two irrationals is
(A) a rational number (B) an irrational number
(C) either A or B (D) neither A nor B

Q.126 The value of $1.\overline{34}+4.\overline{12}$ is
(A) $\frac{133}{99}$ (B) $\frac{371}{90}$
(C) $\frac{5169}{990}$ (D) $\frac{5411}{990}$

Q.127 The value of $4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}$ is

(A) $\frac{40}{31}$ (B) $\frac{4}{9}$

(C) $\frac{1}{8}$ (D) $\frac{31}{40}$

Q.128 The sum of the additive inverse and multiplicative inverse of 2 is

(A) $\frac{3}{2}$ (B) $\frac{-3}{2}$

(C) $\frac{1}{2}$ (D) $\frac{-1}{2}$

Q.129 If $\sqrt{6}=2.449$ then the value of $\frac{3\sqrt{2}}{2\sqrt{3}}$ is close to

(A) 1.225 (B) 0.816

(C) 0.613 (D) 2.449

Q.130 The value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5......}}}}$ is

(A) 0

(B) 5

(C) both (A) and (B)

(D) none of these

Q.131 Arrange the following numbers in descending order. $-2, \frac{4}{-5}, \frac{-11}{20}, \frac{3}{4}$

(A) $\frac{3}{4} > -2 > \frac{-11}{20} > \frac{4}{-5}$

(B) $\frac{3}{4} > \frac{-11}{20} > \frac{-4}{5} > -2$

(C) $\frac{3}{4} > \frac{4}{-5} > -2 > \frac{-11}{20}$

(D) $\frac{3}{4} > \frac{4}{-5} > \frac{-11}{20} > -2$

ANSWER KEY

1.	B	**2.**	D	**3.**	B	**4.**	C
5.	B	**6.**	C	**7.**	A	**8.**	B
9.	B	**10.**	A	**11.**	C	**12.**	A
13.	C	**14.**	A	**15.**	B	**16.**	D
17.	A	**18.**	A	**19.**	D	**20.**	A
21.	C	**22.**	B	**23.**	C	**24.**	B
25.	D	**26.**	A	**27.**	C	**28.**	A
29.	D	**30.**	A	**31.**	D	**32.**	A
33.	B	**34.**	C	**35.**	A	**36.**	D
37.	B	**38.**	A	**39.**	C	**40.**	B
41.	B	**42.**	C	**43.**	D	**44.**	B
45.	A	**46.**	A	**47.**	C	**48.**	B
49.	D	**50.**	C	**51.**	A	**52.**	D
53.	D	**54.**	B	**55.**	A	**56.**	B
57.	A	**58.**	D	**59.**	C	**60.**	C
61.	A	**62.**	D	**63.**	C	**64.**	C
65.	B	**66.**	A	**67.**	B	**68.**	D
69.	A	**70.**	D	**71.**	B	**72.**	D
73.	D	**74.**	A	**75.**	C	**76.**	B
77.	B	**78.**	D	**79.**	A	**80.**	D
81.	A	**82.**	D	**83.**	B	**84.**	C
85.	C	**86.**	A	**87.**	C	**88.**	D
89.	C	**90.**	A	**91.**	D	**92.**	C
93.	A	**94.**	D	**95.**	A	**96.**	B
97.	C	**98.**	D	**99.**	C	**100.**	D
101	A	**102.**	B	**103.**	A	**104.**	D
105.	C	**106.**	B	**107.**	D	**108.**	B
109.	A	**110.**	B	**111.**	B	**112.**	D
113.	A	**114.**	C	**115.**	B	**116.**	B
117.	C	**118.**	D	**119.**	A	**120.**	B
121.	C	**122.**	D	**123.**	C	**124.**	A
125.	C	**126.**	D	**127.**	C	**128.**	B
129.	A	**130.**	C	**131.**	B		

POLYNOMALS

BASIC CONCEPTS AND IMPORTANT RESULTS

1. **Constant :** It is a symbol whose value always remains remains the same, whatever the situation be

For example : 8, −6, π, $\frac{3}{8}$, $\frac{7}{15}$ etc.

2. **Variable :** A symbol which may be assigned different numerical values i.e. changes according to the situation. For example : x, y, z, ax; $2x^2 - 7x + 5$ here x is variable

similarly, area of circle = πr^2 here π is constants and r is variables.

3. **Algebraic Expressions :** A combination of constants and variables, connected by some or all of the operations +, −, × and ÷ is known as an algebraic expression

For example : $5x - 2y$, $7x - 3y + 5z$ ...... etc.

4. **Terms of an Algebric expressions :** The various parts of an algebraic expression that are separated by '+' or '−' sign are called terms.

For example $2x + 3y$ contain 2 terms; $x^3 + 5x^2 - 3x + 5$ contains 4 terms

Various types of algebraic expressions are :

(a) Monomial : An algebraic expression having only one term.

Such as : 5x, 5xyz, $-4x^2$ etc

(b) Binomial : An algebraic expression having two terms.

Such as : $2x + 3$, $5y + 7z$, $8y^2 - 3y$, $4x^2 - 3z$ ... etc.

(c) Trinomial : An algebraic expression having three terms.

Such as : $2x^2 - 3x + 5$, $4x + 5y - 7z$, $9y^2 - 2y + 3x$ ...... etc.

5. **Factor :** Each combination of the constants and variables, which form a term, is called a factor.

For example (i) 7, x and 7x are facters of 7x

(ii) In $-8x^2z$, the numerical facter is −8 and literal (variable) factors are x, z, xz, x^2 and x^2z.

6. **Coefficient :** Any factor of a term is called the coefficient of the remaining term.

For example : (i) In 8x; 8 is coefficient of x & x is coefficient of 8

(ii) In $2x^4 - 5x^3 + \frac{2}{3}x^2 - \sqrt{2}x + 7$, cofficient of x^4, x^3, x^2 and x are 2, −5, $\frac{2}{3}$ and $-\sqrt{2}$ respectively, while 7 is the constant term.

7. **Polynomials :** Algebraic expressions in which the variables involved have only non-negative integral exponents are called polynomials.

8. **Degree of a polynomial :** Highest power of the variable in a polynomial examples :

(i) $3x^7 - 4x^5 + x + 9 \rightarrow$ Degree is 7 (ii) $x^3 - 2x^2y^2 + 3xy + 5y + 4 \rightarrow$ Degree is 4

9. **Constant Polynomial :** A polynomial of degree zero is called a constant polynomial. Examples : 5, −3, $\frac{7}{5}$ etc.

10. **Zero polynomial :** The constant polynomial O is called zero polynomial. Degree of zero polynomial is not defined.

11. Various types of polynomials :

(i) Linear polynomial : A polynomial of degree one. eg. $3x$, $3x + 7$, ...

(ii) Quadratic polynomial : A polynomial of degree two eg. $x^2 - 3x + 7$, $7x^2$, ...

(iii) Cubic polynomial : A polynomial of degree three eg :

$2x^3 - 3x^2 + 7x + 5$, $2x^3 + 3x^2 + 5$, $2x^3 - 5$, $2x^2y - 5xy^2 + 3$, ...

(iv) Biquadratic Polynomial : A polynomial of degree four. Eg :

$3x^4 - 7x^3 + 2x^2 + 5$, $3x^4 - 2x^2 + 5$, $3x^4$, $x^2y^2 + xy^3 + y^4 - 2xy + y^2 + 3$, etc ......

✯ Numbers of terms in a polynomial : Monomial, binomial, Trinomial.

12. Value of a Polynomial : Value of a polynomial $p(x)$ at $x = a$ is $p(a)$

13. Zeroes of a polynomial : Zero of a polynomial $p(x)$ is a number such that $p(a) = 0$.

14. Polynomial Equation : If $p(x)$ is a polynomial then $p(x) = 0$ is a polynomial equation.

Some important observations :

1. A zero of a polynomial need not be zero.
2. 'O' may be a zero of a polynomial.
3. Every linear polynomial has one and only one zero.
4. A polynomial can have more than one zero.
5. A non zero constant polynomial has no zero.
6. Every real number is a zero of the zero polynomial.
7. If the degree of a polynomial is n; the largest number of zeroes it can have is also n.

15. Remainder Theorem : Let $f(x)$ be a polynomial of degree $n \geq 1$ and let a be any real number. When $f(x)$ is divided by $(x - a)$, then the remainder is $f(a)$.

16. Factor Theorem : If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then

(i) $x - a$ is a factor of $p(x)$, if $p(a) = 0$

(ii) $p(a) = 0$, if $x - a$ is a factor of $p(x)$.

(iii) $(x + a)$ is a factor of polynomial $p(x)$ if $p(-a) = 0$

(iv) $(ax - b)$ is a factor of polynomial $p(x)$, if $p\left(\frac{b}{a}\right) = 0$

(v) $(x - a)(x - b)$ are factors of polynomial $p(x)$, if $p(a) = 0$ and $p(b) = 0$.

17. Application of Remainder Theorem in factorisation :

(i) Let the given polynomial be $f(x)$

(ii) Get all positive and negative factors of the constant term.

(iii) Let the first factor be a. Then put $x = a$ in the polynomial $f(x)$. If $f(a) = 0$, then $x - a$ is one of its factors.

(iv) After getting the factor, divide $f(x)$ by the factor obtained. The degree of the quotient will be reduced. If it is 2, factorise it by the methods done already. If it is greater than 2, repeat steps (iii), (iv) and (v) again.

18. **Factors :** A polynomial g(x) is called a factor of the polynomial p(x) if g(x) divides p(x) exactly eg. $(x - y)$ is a factor of $x^3 - y^3$.

19. **Factorisation :** To express a given polynomial as the product of polynomial each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorisation.

Eg. $x^2 - 5x + 6 = (x - 3)(x - 2)$

20. **Methods of Factorisations :**

(a) Common factor

(b) By grouping the terms

(c) Factorisation by making a perfect square.

(d) Difference of two squares.

(e) Quadratic polynomials by splitting the middle term.

(f) Square of a trinomial

(g) Cube of Binomial

(h) Factorisation of sum or difference of cube

(i) Factorsiation of $x^3 + y^3 + z^3 - 3xyz$

(j) Factorsiation of algebraic expressions of the type $x^3 + y^3 + z^3 = 3xyz$ when $x + y + z = 0$

21. **Algebraic Identities :**

(i) $(x + y)^2 = x^2 + y^2 + 2xy$

(ii) $(x - y)^2 = x^2 + y^2 - 2xy$

(iii) $(x^2 - y^2) = (x - y)(x + y)$

(iv) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(v) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

(vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

(viii) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ix) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

(x) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

(xi) If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

(xii) $x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

(xiii) $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

(xiv) $x^8 - y^8 = (x + y)(x - y)(x^2 + y^2)(x^4 + y^4)$

(xv) $x^4 + x^2y^2 + y^4 = (x^2 - xy + y^2)(x^2 + xy + y^2)$ (xvi) $x^4 + y^4 = (x^2 + \sqrt{2}xy + y^2)(x^2 - \sqrt{2}xy + y^2)$

SOLVED PROBLEMS

Ex.1 Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Sol. (i) $4x^2 - 3x + 7$

This expression is a polynomial in one variable x because there is only one variable (x) in the expression.

(ii) $y^2 + \sqrt{2}$

This expression is a polynomial in one variable y because there is only one variable (y) in the expression.

(iii) $3\sqrt{t} + t\sqrt{2}$

The expression is not a polynomial because in the term $3\sqrt{t}$, the exponent of t is $\frac{1}{2}$, which is not a whole number.

(iv) $y + \frac{2}{y}$

This expression is not a polynomial because is the term $\frac{2}{y}$, the exponent of y is -1 which is not a whole number.

(v) $x^{10} + y^3 + t^{50}$

This expression is not a polynomial in one variable because there are three variables (x, y and t) in the expression.

Ex.2 Write the coefficient of x^2 in each of the following :

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$ (iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2} - 1$

Sol. (i) $2 + x^2 + x$ Coefficient of $x^2 = 1$

(ii) $2 - x^2 + x^3$ Coefficient of $x^2 = -1$

(iii) $\frac{\pi}{2}x^2 + x$

Coefficient of $x^2 = \frac{\pi}{2}$

(iv) $\sqrt{2} - 1$

Coefficient of $x^2 = 0$

Ex.3 Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol. One example of a binomial of degree 35 is

$3x^{35} - 4$.

One example of a monomial of degree 100 is $\sqrt{2}\,y^{100}$.

Ex.4 Write the degree of each of the following polynomials :

(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$ (iii) $5t - \sqrt{7}$ (iv) 3

Sol. (i) $5x^3 + 4x^2 + 7x$

Term with the highest power of x = $5x^3$

Exponent of x in this term = 3

$\therefore$ Degree of this polynomial = 3.

(ii) $4 - y^2$

Term with the highest power of y = $-y^2$

Exponent of y in this term = 2

$\therefore$ Degree of this polynomial = 2.

(iii) $5t - \sqrt{7}$

Term with the highest power of t = 5t

Exponent of t in this term = 1

$\therefore$ Degree of this polynomial = 1.

(iv) 3

It is a non-zero constant. So the degree of this polynomial is zero.

Ex5 Classify the following as linear, quadratic and cubic polynomials :

(i) $x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$

(iv) $1 + x$ (v) $3t$ (vi) r^2 (vii) $7x^2$

Sol. (i) Quadratic (ii) Cubic

(iii) Quadratic (iv) Linear

(v) Linear (vi) Quadratic

(vii) Quadratic

Ex.6 Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Sol. Let $f(x) = 5x - 4x^2 + 3$

(i) value of $f(x)$ at $x = 0 = f(0)$

$= 5(0) - 4(0)^2 + 3 = 3$

(ii) value of $f(x)$ at $x = -1 = f(-1)$

$= 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$

(iii) value of $f(x)$ at $x = 2 = f(2)$

$= 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = -3.$

Ex.7 Find $p(0)$, $p(1)$, $p(2)$, for each of the following polynomials :

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$

Sol. (i) $p(y) = y^2 - y + 1$

$\therefore$ $p(0) = (0)^2 - (0) + 1 = 1,$

$p(1) = (1)^2 - (1) + 1 = 1,$ and $p(2) = (2)^2 - (2) + 1 = 4 - 2 + 1 = 3.$

(Rest Try Yourself)

Ex.8 Verify whether the following are zeroes of the polynomial, indicated against them,

(i) $p(x) = 3x + 1,\ x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi,\ x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1,\ x = 1, -1$

Sol. (i) $p(x) = 3x + 1,\ x = -\frac{1}{3}$

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

$\therefore -\frac{1}{3}$ is a zero of $p(x)$. **(Rest Try Yourself)**

Ex.9 Find the zero of the polynomial in each of the following cases :

(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$ (iii) $p(x) = 2x + 5$

Sol. (i) $p(x) = x + 5$

$p(x) = 0$

$\Rightarrow x + 5 = 0 \Rightarrow x = -5$ $\therefore -5$ is zero of the polynomial $p(x)$. **(Rest Try Yourself)**

Ex.10 Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by :

(i) $x + 1$ (ii) $x - \frac{1}{2}$

Sol. (i) $x + 1$

$x + 1 = 0 \Rightarrow x = -1$

$\therefore$ Remainder $= p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

(Rest Try Yourself)

Ex.11 Find the remainder when $x^3 - ax^2 + 6x - a$ divided by $x - a$.

Sol. Let $p(x) = x^3 - ax^2 + 6x - a$

$x - a = 0 \Rightarrow x = a$

$\therefore$ Remainder $= (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

Ex.12 Determine which of the following polynomials, $(x + 1)$ is a factor of :

(i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

Sol. (i) $x^3 + x^2 + x + 1$

Let $p(x) = x^3 + x^2 + x + 1$

The zero of $x + 1$ is $- 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$ **(Rest Try Yourself)**

Ex.13 Use the factor theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$.

Sol. (i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$.

$g(x) = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$

$\therefore$ Zero of $g(x)$ is -1

Now, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0$

$\therefore$ By factor theorem, g(x) is a factor of p(x). **(Rest Try Yourself)**

Ex.14 Find the value of k, if $x - 1$ is a factor of p(x) in each of the following cases :

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

Sol. (i) $p(x) = x^2 + x + k$

If $x - 1$ is a factor of p(x), then $p(1) = 0$

$\Rightarrow (1)^2 + (1) + k = 0 \Rightarrow 1 + 1 + k = 0$

$\Rightarrow 2 + k = 0 \Rightarrow k = -2$ **(Rest Try Yourself)**

Ex.15 Factorise :

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

Sol. (i) $12x^2 - 7x + 1$

$12x^2 - 7x + 1 =$ (i) $12x^2 - 4x - 3x + 1 = 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$ **(Rest Try Yourself)**

Ex.16 Factorise :

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$

Sol. (i) $x^3 - 2x^2 - x + 2$

Let $p(x) = x^3 - 2x^2 - x + 2$

By trial, we find that p(1)

$= (1)^3 - 2(1)^2 - (1) + 2$

$= 1 - 2 - 1 + 2 = 0$

$\therefore$ By factor Theorem $(x - 1)$ is a factor of p(x).

Now, $x^3 - 2x^2 - x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)\{x(x- 2) + 1(x - 2)\}$

$= (x - 1)(x- 2)(x + 1)$ **(Rest Try Yourself)**

Ex.17 Use suitable identities to find the following products :

(i) $(x + 4)(x + 10)$ (ii) $(x + 8)(x - 10)$

Sol. (i) $(x + 4)(x + 10) = x^2 + (4 + 10)x + (4)(10) = x^2 + 14x + 40$ **(Rest Try Yourself)**

Ex.18 Evaluate the following product without multiplying directly :

(i) 103×107 (ii) 95×96

Sol. (i) $103 \times 107 = (100 + 3) \times (100 + 7)$

$= (100)^2 + (3 + 7)(100) + (3)(7)$

$= 10000 + 1000 + 21 = 11021$

Aliter :

$103 \times 107 = (105 - 2) \times (105 + 2) = (105)^2 - (2)^2 = (100 + 5)^2 - 4 = (100)^2 + 2(100)(5) + (5)^2 - 4$

$= 10000 + 1000 + 25 - 4 = 11021.$ **(Rest Try Yourself)**

Ex.19 Factorise the following using appropriate identities :

(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$

Sol. (i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2 = (3x + y)^2 = (3x + y)(3x + y)$

(Rest Try Yourself)

Ex.20 Expand each of the following using suitable identities :

(i) $(x + 2y + 4z)$ (ii) $(2x - y + z)^2$

Sol. (i) $(x + 2y + 4z) = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$ **(Rest Try Yourself)**

Ex.21 Factorise :

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx$

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz = (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(-2x)$

$= \{2x + 3y + (-4z)\}^2 = (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$ **(Rest Try Yourself)**

Ex.22 Write the following cubes in expanded form :

(i) $(2x + 1)^3$ (ii) $(2a - 3b)^3$

Sol. (i) $(2x + 1)^3$

$(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1) = 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$ **(Rest Try Yourself)**

Ex.23 Evaluate the following using suitable identities :

(i) $(99)^3$ (ii) $(102)^3$

Sol. (i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300 = 970299$ **(Rest Try Yourself)**

Ex.24 Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 3(2a)(b)(2a + b) = (2a + b)^3$

$= (2a + b)(2a + b)(2a + b)$ **(Rest Try Yourself)**

Ex.25 Verify : $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Sol. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$ $\quad\Rightarrow\quad$ $x^3 + y^3 = (x + y)\{(x + y)^2 - 3xy\}$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 - xy + y^2$

Ex.26 Factorise : $27y^3 + 125z^3$

Sol. $27y^3 + 125z^3$

$\Rightarrow (3y)^3 + (5z)^3$

$\Rightarrow (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2\}$

$\Rightarrow (3y + 5z)(9y^2 - 15yz + 25z^2)$

Ex.27 Check whether 1 and 2 are zeroes of the polynomial $2x^4 - 6x^3 + 3x^2 + 3x - 2$.

Sol. Let $p(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2 \Rightarrow p(1) = 2 \times (1)^4 - 6(1)^3 + 3(1)^2 + 3(1) - 2$

$= 2 - 6 + 3 + 3 - 2 = 0 \Rightarrow$ 1 is a zero of the given polynomial.

Also, $p(2) = 2 \times 2^4 - 6 \times 2^3 + 3 \times 2^2 + 3 \times 2 - 2 = 32 - 48 + 12 + 6 - 2 = 0$

$\Rightarrow$ 2 is a zero of the given polynomial. $\therefore$ 1 and 2 are zeroes of given polynomial.

Ex.28 Consider the polynomial $p(x) = x^4 + ax^3 - 7x^2 + 8x + b$. Find the values of a and b if p(x) has 0 and –2 as its zeros.

Sol. We have, $p(x) = x^4 + ax^3 - 7x^2 + 8x + b$. $\because$ 0 is a 'zero' of p(x).

$P(0) = 0 \quad \therefore \quad 0 + a \times 0 - 7 \times 0 + 8 \times 0 \times b = 0 \Rightarrow b = 0$ ...(1)

Also as –2 is a zero of p(x) $p(-2) = 0 \Rightarrow (-2)^4 - a(-2)^3 - 7(-2)^2 + 8(-2) + b = 0$

$\Rightarrow 16 - 8a - 28 - 16 + b = 0 \Rightarrow -8a + b = 28 \Rightarrow -8a = 28$ (using b = 0 from equation (1))

$\Rightarrow a = -\frac{28}{8} \Rightarrow a = \frac{-7}{2}$ Hence $a = \frac{-7}{2}$, $b = 0$.

Ex.29 If $p(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $(x - 1)$ and $(x + 1)$ the remainder are respectively 6 and 14. Determine the remainder when p(x) is divided by $(x - 2)$.

Sol. We have $p(x) = x^4 - 2x^3 + 3x^2 - ax + b$.

Let $q(x) = x - 1$ and $r(x) = x + 1$

Zero of q(x) is given by $x - 1 = 0$ i.e., $x = 1$ and zero of r(x) is given by $x + 1 = 0$ i.e., $x = -1$.

By remainder theorem, p(x) when divided by q(x) remainder = p(1).

But according to question remainder = 6 $\therefore$ $p(1) = 6$

$\Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 6 \Rightarrow 1 - 2 + 3 - a + b = 6 \Rightarrow -a + b = 6 - 2 = 4$ ...(1)

Similarly on dividing p(x) by r(x) remainder = p(–1). But according to question it is 14.

$\therefore p(-1) = 14 \Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 14 \Rightarrow 1 + 2 + 3 + a + b = 14$

$\Rightarrow a + b = 14 - 6 = 8$ ...(2)

Adding equations (1) and (2), we get $2b = 12 \Rightarrow b = 6$

Substituting value of b in equations (2), we get $a + 6 = 8 \Rightarrow a = 8 - 6 = 2$

Hence, $p(x) = x^4 - 2x^3 + 3x^2 - 2x + 6$

Now when p(x) is divided by x – 2 (whose zero is 2) the remainder = p(2)

$= 2^4 - 2 \times 2^3 + 3 \times 2^2 - 2 \times 2 + 6 = 16 - 16 + 12 - 4 + 6 = 14$.

Ex.30 If the polynomial $p(x) = 6x^3 + 2x^2 - ax + b$, when divided by $(x + 1)$ leaves remainder 14 and $(x - 1)$ is a factor of p(x). Find a and b.

Sol. We have $p(x) = 6x^3 + 2x^2 - ax + b$.

Given that p(x) when divided by (x + 1) leaves remainder 14.

$\therefore$ By remainder theorem $p(-1) = 14$ ($\because$ zero of x + 1 is –1)

$\Rightarrow 6(-1)^3 + 2(-1)^2 - a(-1) + b = 14 \Rightarrow -6 + 2 + a + b = 14 \Rightarrow a + b = 18$ ...(1)

Also (x – 1) is a factor of p(x) $\therefore$ By factor theorem p(1) = 0

$\Rightarrow 6 \times 1^3 + 2 \times 1^2 - a \cdot 1 + b = 0 \Rightarrow 6 + 2 - a + b = 0 \Rightarrow -a + b = -8$ ...(2)

Adding equations (1) and (2), we get $(a + b) + (-a + b) = 18 + (-8) \Rightarrow 2b = 10 \Rightarrow b = 5$

Substituting b = 5 in equation (1), we get $a + 5 = 18 \Rightarrow a = 13$

Thus the required values of a and b are 13 and 5 respectively.

Ex.31 What should be added to $x^3 - 7x - 16$ so that the resultant polynomial is exactly divisible by $(x^2 - x - 6)$?

Sol. Let, $p(x) = x^3 - 7x - 16$ and $q(x) = x^2 - x - 6$

$= x^2 - 3x + 2x - 6$ (by splitting the middle term)

$= x(x - 3) + 2(x - 3) = (x - 3)(x + 2)$

Let $r(x) = ax + b$ be added to p(x) to make it exactly divisible by q(x).

i.e. $u(x) = p(x) + r(x) = x^3 - 7x - 16 + ax + b$ is divisible by $x^2 - x - 6 = (x - 3)(x + 2)$.

$\Rightarrow$ u(x) is divisible by $(x - 3)$ as well as by $(x + 2)$.

$(x - 3)$ and $(x + 2)$ are factors of u(x).

$\therefore$ $(x - 3)$ is a factor of u(x), therefore by factor theorem, $u(3) = 0$.

$\Rightarrow$ $(3)^3 - 7 \times 3 - 16 + a \times 3 + b = 0$ $\Rightarrow$ $27 - 21 - 16 + 3a + b = 0 \Rightarrow 3a + b = 10$ ...(1)

$\because$ $(x + 2)$ is a factor of u(x), therefore by factor theorem, $u(-2) = 0$.

$\Rightarrow$ $(-2)^3 - 7 \times (-2) - 16 + a(-2) + b = 0$ $\Rightarrow$ $-8 + 14 - 16 - 2a + b = 0 \Rightarrow -2a + b = 10$ ...(2)

Subtracting equation (2) from (1), we get $(3a + b) - (-2a + b) = 10 - 10$

$\Rightarrow$ $3a + b + 2a - b = 0 \Rightarrow 5a = 0$ $\Rightarrow$ $a = 0$

Substituting $a = 0$ is equation (1), we get $3 \times 0 + b = 10 \Rightarrow b = 10$ $\therefore$ $r(x) = 10$ should be added to p(x).

Ex.32 Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

Sol. Let $p(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$

and $q(x) = x^2 - 3x + 2 = x^2 - 2x - x + 2 = x(x - 2) - 1(x - 2) = (x - 1)(x - 2)$

In order to prove that p(x) is divisible by q(x), it is sufficient to prove that p(x) is divisible by $(x - 1)$ and $(x - 2)$.

Zero of $(x - 1)$ is 1, and $p(1) = 2 \times 1^4 - 5 \times 1^3 + 2 \times 1^2 - 1 + 2 = 2 - 5 + 2 - 1 + 2 = 0$

$\therefore$ By factor theorem $(x - 1)$ is a factor of p(x) or p(x) is divisible by $(x - 1)$.

Also zero of $(x - 2)$ is 2, and $p(2) = 2 \times 2^4 - 5 \times 2^3 + 2 \times 2^2 - 2 + 2 = 32 - 40 + 8 - 2 + 2 = 0$

$\therefore$ By factor theorem $(x - 2)$ is a factor of p(x) or p(x) is divisible by $(x - 2)$.

Hence p(x) is divisible by $(x - 1)(x - 2)$ i.e., by $x^2 - 3x + 2$.

Ex.33 Factorise each of the following polynomials :

(i) $9x^2 - 16y^2$ (ii) $27a^2 - 48b^2$ (iii) $x^2 - y^2 + 6y - 9$ (iv) $x^2 - 1 - 2a - a^2$ (v) $x^4 - 625$

Sol. (i) $9x^2 - 16y^2 = (3x)^2 - (4y)^2$

$= (3x + 4y)(3x - 4y)$

(ii) $27a^2 - 48b^2 = 3(9a^2 - 16b^2) = 3[(3a)^2 - (4b)^2] = 3(3a + 4b)(3a - 4b)$

(iii) $x^2 - y^2 + 6y - 9$ $x^2 - (y^2 - 6y + 9) = x^2 - (y^2 - 2 \times y \times 3 + 3^2] = x^2 - (y - 3)^2$

$= (x + y - 3)(x - y + 3)$

(iv) $x^2 - 1 - 2a - a^2 = x^2 - (1 + 2a + a)^2 = x^2 - (1 + a)^2 = (x + 1 + a)(x - 1 - a)$

(v) $x^4 - 625 = (x^2)^2 - (25)^2 = (x^2 + 25)(x^2 - 25) = (x^2 + 25)(x^2 - 5^2) = (x^2 + 25)(x + 5)(x - 5)$

$3[(3a)^2 - (4b)^2] = 3(3a + 4b)(3a - 4b)$

Ex.34 If $2x - 3y = 5$ and $xy = 4$, find the value of $4x^2 + 9y^2$.

Sol. We have, $2x - 3y = 5$ and $xy = 4$.

Also we know that $(2x - 3y)^2 = (2x)^2 + (3y)^2 - 2 \times 2x \times 3y$

$\Rightarrow$ $(2x - 3y)^2 = 4x^2 + 9y^2 - 12xy$ $\Rightarrow$ $5^2 = 4x^2 + 9y^2 - 12 \times 4$

$\Rightarrow$ $4x^2 + 9y^2 = 25 + 48 = 73$

$\therefore$ $4x^2 + 9y^2 = 73$

Ex.35 If $x + \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$.

Sol. We have, $x + \frac{1}{x} = 3$

$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 = 3^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 9 \quad \Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$

Ex.36 If $9x^2 + y^2 = 10$ and $xy = -1$, find the value of $3x + y$.

Sol. We have, $9x^2 + y^2 = 10$ and $xy = -1$

Also we know that $(3x + y)^2 = (3x)^2 + y^2 + 2 \times 3x \times y$

$\Rightarrow (3x + y)^2 = 9x^2 + y^2 + 6xy \quad \Rightarrow (3x + y)^2 = 10 - 6 \quad \Rightarrow (3x + y)^2 = 4 \quad \therefore 3x + y = \pm 2$

Ex.37 If $a + b + c = 6$, $ab + bc + ca = 11$, find the value of $a^2 + b^2 + c^2$.

Sol. We have

$(a + b + c) = 6 \quad ...(1) \quad$ and $\quad ab + bc + ca = 11 \quad ...(2)$

Also we know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca) = (6)^2 - 2 \times 11$

Q (using equations (1) and (2)) $= 36 - 22 = 14 \quad \therefore a^2 + b^2 + c^2 = 14$

Ex.38 If $4a^2 + 9b^2 + 25c^2 = 38$ and $6ab + 15bc + 10ac = -19$, find the value of $2a + 3b + 5c$.

Sol. We have,

$4a^2 + 9b^2 + 25c^2 = 38 \quad ...(1)$

and $\quad 6ab + 15bc + 10ac = -19 \quad ...(2)$

Also,

$(2a + 3b + 5c)^2 = (2a)^2 + (3b)^2 + (5c)^2 + 2(2a) \times (3b) + 2(3b)(5c) + 2(5c)(2a)$

$\Rightarrow (2a + 3b + 5c)^2 = 4a^2 + 9b^2 + 25c^2 + 12ab + 30bc + 20ca$

$\Rightarrow (2a + 3b + 5c)^2 = (4a^2 + 9b^2 + 25c^2) + 2(6ab + 15bc + 10ca)$

$\Rightarrow (2a + 3b + 5c)^2 = 38 + 2(-19)$ (using equation (1) and (2))

$\Rightarrow (2a + 3b + 5c)^2 = 0$

$\therefore 2a + 3b + 5c = 0$

Ex.39 If $a + b + c = 8$, $ab + bc + ca = 28$, find the value of $a^3 + b^3 + c^3 - 3abc$.

Sol. We have,

$a + b + c = 8 \quad ...(1) \quad$ and $\quad ab + bc + ca + 28 \quad ...(2)$

$\therefore \quad (a + b + c)^2 = (8)^2 \quad \Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + ca = 64$

$\Rightarrow \quad (a^2 + b^2 + c^2) + 2(ab + bc + ca) = 64$

$\Rightarrow \quad (a^2 + b^2 + c^2) + 2 \times 28 = 64$ (using equation (2))

$\Rightarrow \quad a^2 + b^2 + c^2 = 64 - 56 = 8 \quad ...(3)$

Now, $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (8) \times [8 - 28]$ (using equations (1), (2) and (3)) $= 8 \times (-20) = -160$.

Ex.40 Give possible expressions for the length and breadth of each of the following rectangles in which their areas are given :

(i) Area = $25a^2 - 35a + 12$ (ii) Area = $35y^2 + 13y - 12$.

Sol. (i) Area = $25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12 = 5a\,(5a - 4) - 3\,(5a - 4) = (5a - 4)\,(5a - 3)$

$\therefore$ Some possible values of length and breadth of the rectangle are

Length	**Breadth**
1	$25a^2 - 35a + 12$
$5a - 4$	$5a - 3$
$5a - 3$	$5a - 4$
$25a^2 - 35a + 12$	1

(ii) Area = $35y^2 + 13y - 12 = 35y^2 + 35y - 12y - 12 = 35y\,(y + 1) - 12\,(y + 1) = (y + 1)\,(35y - 12)$

$\therefore$ Some possible values of length and breadth of rectangle are as follows –

Length	**Breadth**
1	$35y^2 - 13y - 12$
$y + 1$	$35y - 12$
$35y - 12$	$y + 1$
$35y^2 + 13y - 12$	1

Ex.41 What are the possible expressions for the dimensions of the cuboids whose volumes are given below:

(i) Volume = $3x^2 - 12x$ (ii) Volume = $12ky^2 + 80ky - 20k$

Sol. (i) Volume = $3x^2 - 12x = 3x\,(x - 4)$

$\therefore$ Some of the dimensions of the cuboids of given volume can have expressions as given below –

Length	**Breadth**	**Height**
3	x	$x - 4$
x	3	$x - 4$
$x - 4$	3	x
$x - 4$	x	3
3	$x - 4$	x
x	$x - 4$	3

(ii) Volume = $12ky^2 + 80ky - 20k = 4k\,(3y^2 + 20y - 5)$ [or we factorise $3y^2 + 20y - 5$]

$\therefore$ Some of the dimensions of the cuboids of given volume can have expressions as given below

Length	**Breadth**	**Height**
4	k	$3y^2 + 20y - 5$
k	4	$3y^2 + 20y - 5$
4	$3y^2 + 20y - 5$	k
k	$3y^2 + 20y - 5$	4
$3y^2 + 20y - 5$	4	k
$3y^2 + 20y - 5$	k	4

EXERCISE – I | UNSOLVED PROBLEMS

Q.1 Write the coefficient of :
(i) x^2 in $3x^3 - 5x^2 + 7$
(ii) xy in 8 xyz
(iii) $-y$ in $2y^2 - 6y + 2$
(iv) x^0 in $3x + 7$

Q.2 Find which of the following algebraic expression is a polynomial.
(i) $3x^2 - 5x$ (ii) $x + \frac{1}{x}$
(iii) $\sqrt{y} - 8$ (iv) $z^5 - \sqrt[3]{z} + 8$

Q.3 Find the degree of the polynomial :
(i) $5x^2 - 6x^3 + 8x^7 + 6x^2$
(ii) $2y^{12} + 3y^{10} - y^{15} + y + 3$
(iii) x (iv) 8

Q.4 Write the coefficient of x^2 in each of the following :
(i) $2 + x^2 + x$ (ii) $5 - x^2 + x^3$
(iii) $\frac{1}{2}x^2 + x$ (iv) $\sqrt{5}\,x - 1$

Q.5 Classify the following as linear, quadratic and cubic polynomials :
(i) $x^2 + x$ (ii) $x - x^3$
(iii) $1 + x$ (iv) 3t
(v) r^2 (vi) $7x^3$
(vii) $y + y^2 + 5$ (viii) 3xyz

Q.6 Find q(0), q(1) and q(2) for each of the following polynomials :
(i) $q(x) = x^2 + 3x$
(ii) $q(y) = 2 + y + 2y^2 - 5y^3$
(iii) $q(t) = t^3$

Q.7 Check whether 0 and 3 are zeroes of the polynomial $x^2 - 3x$.

Q.8 Show that 3 is a zero of the polynomial $x^3 - 8x^2 + 8x + 21$.

Q.9 Which of the number 1, –1, and –3 are zeroes of the polynomial $2x^4 + 9x^3 + 11x^2 + 4x - 6$.

Q.10 Verify whether the indicated numbers are zeroes (roots) of the polynomial corresponding to them in the following cases:
(i) $p(x) = 3x + 1,\ x = -\frac{1}{3}$
(ii) $p(x) = (x + 1)(x - 2),\ x = -1, 2$
(iii) $p(x) = x^2,\ x = 0$
(iv) $p(x) = \ell x + m,\ x = -\frac{m}{\ell}$
(v) $p(x) = 2x + 1,\ x = \frac{1}{2}$

Q.11 Find the zero of the polynomial in each of the following cases :
(i) $p(x) = x + 5$
(ii) $p(x) = 2x + 5$
(iii) $p(x) = 3x - 2$

Q.12 Find the integral zeroes of the polynomial $x^3 + x^2 + x - 3$.

Q.13 Find the zeroes of the quadratic polynomial $6x^2 - 13x + 6$ and verify the relation between the zeroes and its coefficients.

Q.14 Find the zeroes of the quadratic polynomial $4x^2 - 9$ and verify the relation between the zeroes and its coefficients.

Q.15 Find the remainder when $4x^3 - 3x^2 + 2x - 4$ is divided by :
(a) $x - 1$ (b) $x + 2$ (c) $x + \frac{1}{2}$

Q.16 Find the remainder when the polynomial $f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$ is divided by $x + 2$.

Q.17 If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, find the value of a.

Q.18 Let R_1 and R_2 are the remainder when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$, find the value of a.

Q.19 If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x - 1$ and $x + 1$, the remainder are respectivley 5 and 19. Determine the remainder when f(x) is divided by $(x - 2)$.

Q.20 Using factor theorem, factorize the polynomial $x^3 - 6x^2 + 11x - 6$.

Q.21 Using factor theorem, factorize the polynomial $x^4 + x^3 - 7x^2 - x + 6$.

Q.22 Factorize, $2x^4 + x^3 - 14x^2 - 19x - 6$.

Q.23 The coefficient of x^2 in $(px^2 + 4x + r) \times (4x^2 - 3qx - 5)$ is ....

Q.24 The value of $\{x+(y-z)\}^2 - \{y-(z+x)\}^2 + \{z-(x+y)\}^2$ when $x = 1$, $y = -2$, $c = 3$ is ....

Q.25 $\frac{5m^2 - 4m - 2}{5m^2 - 4m} = \frac{7}{2}$ then the value of m ....

Q.26 $abx^2 + (a^2 - b^2)x - ab$ when expressed in linear factors is equal to ....

Q.27 $x^2(y - z) + y^2(z-x) + z^2(x-y) = N(x-y)(y-z)(z-x)$, then N must have the value

Q.28 $4x^2 - 2bx + ab - a^2$ when expressed in linear factors is equal to ....

(Write True or False Q.29 to 36)

Q.29 When $14x^3 - 3x^2 + 4x + 2$ is divided by $2x-1$, the remainder is 5.

Q.30 $4x^2 - 8x + 15$ is exactly divisible by $2x-1$.

Q.31 $x^2 - 5x + 6$ cannot be written as a product of two linear factors.

Q.32 $x^3 - 3x^2y + 3xy^2 - y^3$ is exactly divisible by $x - y$.

Q.33 $(2a + b)^2 - (2b + a)^2 = 3(a^2 - b^2)$ is an identity

Q.34 $9 + 4\sqrt{5}$ and $9 - 4\sqrt{5}$ are reciprocal numbers.

Q.35 $(x^2 + 1)^4 = (x^4 - 6x^2 + 1)^2 + 16(x^3 - x)^2$ is an identity.

Q.36 $3x + [4x - \{3x + (4x-5)+3\} - 5x]$ is independent of x.

Q.37 Factorize the following expression : $6x^2 - 5x - 6$

Q.38 Factorize each of the following expressions

(i) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$

(ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

(iii) $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

Q.39 Factorize the following by splitting the middle term : $\frac{1}{3}x^2 - 2x - 9$

Q.40 Factorize the following trinomial by splitting the middle term : $8a^3 - 2a^2b - 15ab^2$

Q.41 Factorize each of the following expressions by splitting the middle term :

(i) $9(x - 2y)^2 - 4(x - 2y) - 13$

(ii) $2(x + y)^2 - 9(x + y) - 5$

(iii) $8(a + 1)^2 + 2(a + 1)(b + 2) - 15(b + 2)^2$

Q.42 Factorize : $(x - y)^3 + (y - z)^3 + (z - x)^3$

Q.43 Factorize : $(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3$

Q.44 Simplify : $\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$

Q.45 Find the value of $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$.

Q.46 Factorize $27x^3 + 64y^3$

Q.47 Factorize $a^3 + 3a^2b + 3ab^2 + b^3 - 8$

Q.48 Factorize : $a^3 - 0.216$

Q.49 Factorize : (i) $(x+1)^3 - (x-1)^3$

(ii) $8(x + y)^3 - 27(x-y)^3$

Q.50 Factorize : (i) $x^6 - y^6$ (ii) $x^{12} - y^{12}$

Q.51 Prove that :

$$\frac{0.87 \times 0.87 \times 0.87 + 0.13 \times 0.13 \times 0.13}{0.87 \times 0.87 - 0.87 \times 0.13 + 0.13 \times 0.13} = 1$$

Q.52 Factorize : $8x^3 + 27y^3 + z^3 - 18\,xyz$

Q.53 Factorize :
$(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a)$

Q.54 Resolve $a^3 - b^3 + 1 + 3ab$ into factors

Q.55 Factorize : $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$

Q.56 Prove that :
$a^3 + b^3 + c^3 - 3abc$
$= \frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$

Q.57 Find the remainder when $4x^3 - 3x^2 + 2x - 4$ is divided by

(a) $x - 1$ (b) $x + 2$ (c) $x + \frac{1}{2}$

Q.58 Determine the remainder when the polynomial $p(x) = x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$.

Q.59 Find the remainder when the polynomial $f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$ is divided by $x + 2$

Q.60 Find the remainder when
$p(x) = 4x^3 - 12x^2 + 14x - 3$ is divided by
$g(x) = x - \frac{1}{2}$

Q.61 If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x-3)$, find the value of a.

Q.62 Let R_1 and R_2 are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$, find the value of a.

Q.63 Use the factor theorem to determine whether $x - 1$ is a factor of
(a) $x^3 + 8x^2 - 7x - 2$
(b) $2\sqrt{2}\,x^3 + 5\sqrt{2}\,x^2 - 7\sqrt{2}$
(c) $8x^4 + 12x^3 - 18x + 14$

Q.64 Factorize each of the following expression, given that ;
$x^3 + 13x^2 + 32x + 20$. $(x+2)$ is a factor.

Q.65 Factorize ;
$x^3 - 23x^2 + 142x - 120$

Q.66 Show that $(x - 3)$ is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$

Q.67 Show that $(x - 1)$ is a factor of $x^{10} - 1$ and also of $x^{11} - 1$.

Q.68 Show that $x + 1$ and $2x - 3$ are factors of $2x^3 - 9x^2 + x + 12$.

Q.69 Find the value of k, if $x + 3$ is a factor of $3x^2 + kx + 6$.

Q.70 If $ax^3 + bx^2 + x - 6$ has $x + 2$ as a factor and leaves a remainder 4 when divided by $(x - 2)$, find the values of a and b.

Q.71 If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

Q.72 If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, show that $a + c + e = b + d = 0$.

Q.73 Using factor theorem, show that $a - b$, $b - c$ and $c - a$ are the factors of $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

Q.74 Factorize $x^2 + 4 + 9z^2 + 4x - 6xz - 12z$

Q.75 Using factor theorem, factorize the polynomial $x^3 - 6x^2 + 11x - 6$.

Q.76 Using factor theorem, factorize the polynomial $x^4 + x^3 - 7x^2 - x +$

Q.77 Factorize, $2x^4 + x^3 - 14x^2 - 19x - 6$

Q.78 Factorize, $9z^3 - 27z^2 - 100z + 300$, if it is given that $(3z+10)$ is a factor of it.

Q.79 Simplify : $\frac{4x-2}{x^2-x-2} + \frac{3}{2x^2-7x+6} - \frac{8x+3}{2x^2-x-3}$

Q.80 Establish the identity
$\frac{6x^2+11x-8}{3x-2} = (2x+5) + \frac{2}{3x-2}$

Q.81 If $\left(x+\frac{1}{x}\right) = 5$, find the value of
$\left(x^3+\frac{1}{x^3}\right) - 5\left[x^2+\frac{1}{x^2}\right] + 5$

Q.82 Find the value of the polynomial $5x - 4x^2 + 3$ at: (i) $x = 0$ (ii) $x = -1$

Q.83 Find the zero of the polynomial in each of the following cases :
(i) $p(x) = x + 5$
(ii) $p(x) = 2x + 5$
(iii) $p(x) = 3x - 2$

ANSWER KEY

1. (i) −5 (ii) 8z (iii) 6 (iv) Since $x^0 = 1$, Therefore $3x + 7 = 3x + 7x^0 \Rightarrow$ coefficient of x^0 is 7.

2. (i) polynomial (ii) not a polynomial (iii) not a polynomial (iv) not a polynomial.

3. (i) 7 (ii) 15 (iii) 1 (iv) 0

4. (i) 1 (ii) −1 (iii) $\frac{1}{2}$ (iv) 0

5. (i) quadratic polynomial (ii) cubic polynomial (iii) linear polynomial (iv) linear polynomial (v) quadratic polynomial (vi) cubic polynomial (vii) quadratic polynomial (viii)cubic polynomial.

6. (i) 0,4,10 (ii) 2, 0, −28 (iii) 0, 1, 8

7. Yes, 0 and 3 are both zeroes of the polynomial $x^2 - 3x$

9. 1 and −1 are not zeroes of f(x) whereas −3 is a zero of f(x)

10. (i) Yes, (ii) Yes, (iii) Yes, (iv) Yes, (v) No

11. (i) −5 (ii) $\frac{-5}{2}$ (iii) $\frac{2}{3}$ **12.** 1

13. $\frac{2}{3}$ and $\frac{3}{2}$ **14.** $\frac{3}{2}$ and $-\frac{3}{2}$.

15. (a) − 1 (b) − 52 (c) $\frac{-25}{4}$ **16.** 92

17. − 1 **18.** a = 2 **19.** 10

20. $(x - 1)(x - 2)(x - 3)$

21. $(x - 1)(x + 1)(x - 2)(x + 3)$

22. $(x + 1)(x + 2)(x - 3)(2x + 1)$

23. $-5p - 12q + 4r$

24. − 4 **25.** 2/5

26. $(bx + a)(ax - b)$ **27.** N = −1

28. $(2x - a)(2x - b + a)$ **29.** True

30. False **31.** False **32.** True

33. True **34.** True **35.** True

36. False **37.** $(2x - 3)(3x + 2)$

38. (i) $(\sqrt{3}x + 2)(x + 3\sqrt{3})$
(ii) $(\sqrt{3}x + 2)(4x - \sqrt{3})$
(iii) $(x - \sqrt{2})(7\sqrt{2}x + 4)$

39. $(x-9)\left(\frac{1}{3}x+1\right)$

40. $a(2a - 3b)(4a + 5b)$

41. (i) $(x - 2y + 1)(9x - 18y - 13)$
(ii) $(x + y - 5)(2x + 2y + 1)$.
(iii) $(2a + 3b + 8)(4a - 5b - 6)$

42. $3(x-y)(y - z)(z-x)$

43. $3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

44. $(a + b)(b + c)(c + a)$ **45.** 0

46. $(3x + 4y)(9x^2 - 12xy + 16y^2)$

47. $(a + b - 2)(a^2 + 2ab + b^2 + 2a + 2b + 4)$

48. $(a-0.6)(a^2 + 0.6a + 0.36)$

49. (i) $2(3x^2+1)$ (ii) $(-x + 5y)(-x^2 - 2xy + 11y^2)$

50. (i) $(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$
(ii) $(x-y)(x+y)(x^2+y^2)(x^4+y^4-x^2y^2)$

51. 1

52. $(2x+3y+z)\{(4x^2 + 9y^2 + z^2 - 6xy - 3yz - 2zx\}$

53. $2(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

54. $(a-b+1)(a^2 + b^2 + ab - a + b + 1)$

55. $(\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ac)$

56. $\frac{1}{2}(a + b + c)\{(a - b)^2 + (b-c)^2 + (c-a)^2\}$

57. (a)− 1 (b) − 52 (c) $\frac{-25}{4}$

58. p(1) = 1 **59.** 92 **60.** $p\left(\frac{1}{2}\right) = \frac{3}{2}$

61. − 1 **62.** a = 2

63. (a) yes (b) yes (c) no

64. $(x+2)(x + 10)(x + 1)$

65. $(x - 1)(x-10)(x - 12)$

66. $x^3 - 3x^2 + 4x - 12$. **67.** yes

68. yes **69.** 11 **70.** 2.

71. p = r **72.** 0

74. $(x + 2 - 3z)(x + 2 - 3z)$

75. $(x - 1)(x - 2)(x-3)$

76. $(x-1)(x+1)(x-2)(x+3)$

77. $(x + 1)(x + 2)(x - 3)(2x+1)$

78. $(3z + 10)(3z-10)(z-3)$

79. $\frac{15}{(x-2)(x+1)(2x-3)}$ **81.** 0

82. (i) 3 (ii)−6

83. (i) −5 (ii) $\frac{-5}{2}$ (iii) $\frac{2}{3}$

EXERCISE – II | SCHOOL EXAM/BOARD

FACTORIZE EACH OF THE FOLLOWING EXPRESSIONS

Q.1 $x^2 - x - 42$.

Q.2 $1 + 2x + x^2$.

Q.3 $6 - 5y - y^2$.

Q.4 $x^2 - 9ax + 18a^2$.

Q.5 $a^2 + 46a + 205$.

Q.6 $k^2 - 26k + 133$.

Q.7 $ab + ac - b^2 - bc$.

Q.8 $2xy - x + z - 2zy$.

Q.9 $100 - 9p^2$.

Q.10. $p^4 - 81q^4$.

Q.11 $\frac{1}{5}x^2 + 2x - 15$.

Q.12 $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$.

Q.13 $6xy + 6 - 9y - 4x$.

Q.14 $1 - 256x^4$.

Q.15 $x^4 - (2y - 3z)^2$.

Q.16 $(2a + 1)^2 - 9b^4$.

Q.17 $24\sqrt{3}x^3 - 125y^3$.

Q.18 $125\ a^3 + \frac{b^3}{27}$.

Q.19 $a^6x^4 - a^4x^6$.

Q.20 If one of the factors of $x^2 + x - 20$ is $(x + 5)$, find other factor.

Q.21 Find the positive squares root of $36x^2 + 60x + 25$.

Q.22 Simplify :

(i) $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$ (ii) $\sqrt{2}x^2 + 3x + \sqrt{2}$.

Q.23 Factorise : $5(a + 2b)^2 - 7(a + 2b) + 2$.

Q.24 Factorise : $125(x - y)^3 + (5y - 3z)^3 + (3z - 5x)^3$.

Q.25 Factorise : $27x^3 + y^3 + z^3 - 9xyz$.

Q.26 Find the product : (i) $(x + 2)(x + 9)$ (ii) $(x + 8)(x - 2)$ (iii) $(z - 3)(z - 5)$ (iv) $(z^2 + 4)(z^2 - 5)$

Q.27 Evaluate the following : (i) 103×105 (ii) 98×99 (iii) 104×95

Q.28 Write : $\left(\frac{3}{2}x + 1\right)^3$ in expand form.

Q.29 Write : $\left(x - \frac{2}{3}y\right)^3$ is expand form.

Q.30 Write the expansions of each of the following : (i) $(9x + 2y + z)^2$ (ii) $(3x - 2y - z)^2$

Q.31 Simplify : $(2a + b + c)^2 + (2a - b - c)^2$

Q.32 Write the expansion of the following : (i) $(2x + 3y)^3$ (ii) $(p - yz)^3$.

Q.33 Find the value of $27x^3 + 8y^3$ if $3x + 2y = 20$ and $xy = \frac{14}{9}$.

Q.34 Find the value of $a^3 - 27b^3$ if $a - 3b = -6$ and $ab = -10$.

Q.35 Evaluate : (i) $(101)^3$ (ii) $(399)^3$.

Q.36 Find the product of following :

(i) $(x + 3)(x^2 - 3x + 9)$

(ii) $(5a - 3b)(25a^2 + 15ab + 9b^2)$

Q.37 Factorize :

(i) $1 - 27z^3$ (ii) $250x^3 - 16y^3$

(iii) $xy^3 + 729x^4$

Q.38 If $x + y + z = 8$ and $xy + yz + zx = 20$, find the value of $x^3 + y^3 + z^3 - 3xyz$.

Q.39 If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$, find the value of $a^3 + b^3 + c^3 - 3abc$.

Q.40 If $p + q + r = 1$ and $pq + qr + pr = -1$ and $pqr = -1$, find the value of $p^3 + q^3 + r^3$.

Q.41 Factorize : $(x - y)^3 + (y - z)^3 + (z - x)^3$.

Q.42 Find the value of : $(25)^3 - (29)^3 + (4)^3$.

Q.43 With out actual division, prove that $a^4 + 2a^3 - 2a^2 + 2a - 3$ is exactly divisible $a^2 + 2a - 3$.

Q.44 If $(x + 1)$ and $(x - 1)$ are the factors of $mx^3 + x^2 - 2x + n$, find the value of m and n.

Q.45 The polynomials $kx^3 + 3x^2 - 3$ and $2x^3 - 5x + k$ when divided by $(x - 4)$ leave the same remainder or in each case. Find the value of k.

Q.46 What must be added to $x^3 - 3x^2 - 12x + 19$ so that result is exactly divisible by $x^2 + x - 6$?

Q.47 What must be subtrated from $x^3 - 6x^2 - 15x + 80$ so that result is exactly divisible by $x^2 + x - 12$?

Q.48 Using factor theorem, factorise the polynomial $x^4 - 2x^3 - 7x^2 + 8x + 12$.

Q.49 Let A and B are the remainders when the polynomial $y^3 + 2y^2 - 5ay - 7$ and $y^3 + ay^2 - 12y + 6$ are divided by $y + 1$ and $y - 2$ respectively. If $2A + B = 6$, find the value of a.

Q.50 Simplify : $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$.

Q.51 Show that if a + b is not zero, then the equation $a(x - a) = 2ab - b(x - b)$ has a solution $x = a + b$.

Q.52 Show that if $2(a^2 + b^2) = (a + b)^2$, then $a = b$.

Q.53 Find the value of :

(i) $x^3 + y^3 - 12xy + 64$ when $x + y = -4$

(ii) $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$.

(iii) $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ when $a + b + c = 3x$.

Q.54 Prove that $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

Q.55 Prove that $(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = 2(a^3 + b^3 + c^3 - 3abc)$

Q.56 If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, show that $a + c + e = b + d = 0$.

Q.57 Using factor theorem, show that $(x + y)$, $(y + z)$ and $(z + x)$ are the factors of $(x + y - z)^3 - (x^3 + y^3 + z^3)$.

Q.58 If $(3x - 1)^4 = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, then find the value of $a_4 + 3a_3 + 9a_2 + 27a_1 + 81a_0$.

ANSWER KEY

1. $[(x + 6)(x - 7)]$ **2.** $[(x + 1)^2]$
3. $(y - 1)(y + 6)$ **4.** $(x - 6a)(x - 3a)$
5. $(a + 41)(a + 5)$ **6.** $(k - 7)(k - 19)$
7. $(a - b)(b + c)$ **8.** $(x - z)(2y - 1)$
9. $(10 - 3p)(10 + 3p)$
10. $(p + 3q)(p - 3q)(p^2 + 9q^2)$
11. $\frac{1}{5}(x + 15)(x - 5)$
12. $(x - \sqrt{2})(7\sqrt{2}x + 4)$ **13.** $(3y - 2)(2x - 3)$
14. $(1 - 4x)(1 + 4x)(1 + 16x^2)$
15. $(x^2 - 2y + 3z)(x^2 + 2y - 3z)$
16. $(2a + 1 - 3b^2)(2a + 1 + 3b^2)$
17. $(2\sqrt{3}x - 5y)(12x^2 + 10\sqrt{3}xy + 25y^2)$
18. $(5a + \frac{b}{3})(25a^2 - \frac{5}{3}ab + \frac{b^2}{9})$
19. $a^4x^4(a - x)(a + x)$ **20.** $(x - 4)$
21. $(6x + 5)$
22. (i) $(\sqrt{2}a + \sqrt{3}b)$, (ii) $(\sqrt{2}x + 1)(x + \sqrt{2})$
23. $(5a + 10b - 2)(a + 2b - 1)$
24. $3(5x - 5y)(5y - 3z)(3z - 5x)$
25. $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$
26. (i) $x^2 + 11x + 18$, (ii) $x^2 + 6x - 16$, (iii) $z - 8z + 5$, (iv) $z^4 - z^2 - 20$
27. (i) 10815, (ii) 9702, (iii) 9880
28. $\left[\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1\right]$
29. $\left[x^3 - 2x^2y + \frac{4xy^2}{3} - \frac{8}{27}y^3\right]$
30. (i) $81x^2 + 4y^2 + z^2 + 36xy + 4yz + 18zx$, (ii) $9x^2 + 4y^2 + z^2 - 12xy + 4yz - 6xz$
31. $[8a^2 + 2b^2 + 2c^2 + 4bc]$
32. (i) $8x^3 + 27y^3 + 36x^2y + 54xy^2$, (ii) $p^3 - y^3z^3 - 3p^2yz + 3py^2z^2$
33. [7440]
34. [324]
35. (i) 1030301, (ii) 63521199
36. (i) $x^3 + 27$, (ii) $[125a^3 - 27b^3]$
37. (i) $(1 - 3z)(1 + 3z + 9z^2)$, (ii) $2(5x - 2y)(25x^2 + 10xy + 4y^2)$, (iii) $x(y + 9x)(y^2 + 81x - 9xy)$
38. [32] **39.** [108]
40. [1]
41. $[3(x - y)(y - z)(z - x)]$
42. [−8700] **44.** [m = 2, n = −1]
45. [k = 1] **46.** [2x + 5]
47. [4x − 4]
48. $(x + 1)(x + 2)(x - 2)(x - 3)$
49. [a = 2] **50.** $8a^3$
53. (i) 0 (ii) 0 (iii) 0 **58.** [0]

EXERCISE – III OLYMPIAD QUESTIONS

Q.1 If $x + \frac{1}{x} = a + b$ and $x - \frac{1}{x} = a - b$ then
(A) ab = 1 (B) a = b
(C) ab = 2 (D) a + b = 0

Q.2 What must be added to $x^3 + 3x - 8$ to get $3x^3 + x^2 + 6$?
(A) $2x^3 + x^2 - 3x + 14$ (B) $2x^2 + x^2 + 14$
(C) $2x^3 + x^2 - 6x - 14$ (D) None of these

Q.3 If $\left(a+\frac{1}{a}\right)^2 = b$ then $a^3 + \frac{1}{a^3}$ is equal to
(A) b^3 (B) $b^{\frac{3}{2}}$
(C) $b^{\frac{3}{2}} - 3b^{\frac{1}{2}}$ (D) $b^{\frac{3}{2}} + 3b^{\frac{1}{2}}$

Q.4 The product of two factors with unlike signs is ______
(A) positive (B) negative
(C) cannot be determined
(D) none of these

Q.5 Subtract $x^3 - xy^2 + 5x^2y - y^3$ from $-y^3 - 6x^2y - xy^2 + x^3$.
(A) $2y^3 - 8x^2y + 3xy^2 - 2x^3$
(B) $2y^3 - 2xy^2 - x^2y - 2y^3$
(C) $-11x^2y$ (D) none of these

Q.6 The real factors of $x^2 + 4$ are
(A) $(x^2 + 2)(x^2 - 2)$ (B) $(x + 2)(x - 2)$
(C) does not exist (D) none of these

Q.7 What must be subtracted from $x^3 - 3x^2 + 5x - 1$ to get $2x^3 + x^2 - 4x + 2$?
(A) $-x^3 + 4x^2 - 9x + 3$ (B) $x^3 + 4x^2 - 9x + 3$
(C) $x^3 - 4x^2 + 9x - 3$ (D) $-x^3 - 4x^2 + 9x - 3$

Q.8 The factors of $x^4 + y^4 + x^2y^2$ are
(A) $(x^2 + y^2)(x^2 + y^2 - xy)$
(B) $(x^2 + y^2)(x^2 - y^2)$
(C) $(x^2 + y^2 + xy)(x^2 + y^2 - xy)$
(D) factorization is not possible

Q.9 The factors of $15x^2 - 26x + 8$ are
(A) $(3x - 4)(5x + 2)$ (B) $(3x - 4)(5x - 2)$
(C) $(3x + 4)(5x - 2)$ (D) $(3x + 4)(5x + 2)$

Q.10 One of the factors of $a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$ is
(A) a – b (B) b – c
(C) c – a (D) all the above

Q.11 One of the factors of $a^3 + 8b^3 - 64c^3 + 24abc$ is
(A) a + 2b – 4c (B) a – 2b + 4c
(C) a + 2b + 4c (D) a – 2b – 4c

Q.12 Divide $(-56\ mnp^2)$ by $(7\ mnp)$.
(A) –8 p (B) 8 mnp (C) 8 p (D) None

Q.13 The product of $(x^2 + 3x + 5)$ and $(x^2 - 1)$ is
(A) $x^4 + 3x^3 - 4x^2 - 3x - 5$
(B) $x^4 + 3x^3 + 4x^2 - 3x - 5$
(C) $x^4 + 3x^3 + 4x^2 + 3x - 5$
(D) none of these

Q.14 If quotient = $3x^2 - 2x + 1$, remainder = $2x - 5$ and divisor = $x + 2$ then the dividend is
(A) $3x^3 - 4x^2 + x - 3$ (B) $3x^3 - 4x^2 - x + 3$
(C) $3x^3 + 4x^2 - x + 3$ (D) $3x^3 + 4x^2 - x - 3$

Q.15 If $(x - 2)$ is one factor of $x^2 + ax - 6 = 0$ and $x^2 - 9x + b = 0$ then a + b = _____ .
(A) 15 (B) 13 (C) 11 (D) 10

Q.16 Factors of $x^3 - 3x^2 + 3x + 7$
(A) $(x + 1)(x^2 - 4x + 7)$
(B) $(x - 1)(x^2 + 4x + 7)$
(C) $(x + 1)(x^2 + 4x + 7)$
(D) $(x - 1)(x^2 - 4x + 7)$

Q.17 The value of $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$
(A) $6a^3$ (B) $8a^3$ (C) $10a^3$ (D) $12a^3$

Q.18 Factorise $x^2 + 3\sqrt{2}x + 4$
(A) $(x + 2\sqrt{2})(x + \sqrt{2})$ (B) $(x + 2\sqrt{2})(x - \sqrt{2})$
(C) $(x - 2\sqrt{2})(x + \sqrt{2})$ (D) $(x + 2\sqrt{2})(x - \sqrt{2})$

Q.19 The remainder obtained when $t^6 + 3t^2 + 10$ is divided by $t^3 + 1$ is
(A) $t^2 - 11$ (B) $t^3 - 1$
(C) $3t^2 + 11$ (D) none

Q.20 The value of the product $(3x^2 - 5x + 6)$ and $(-8x^3)$ when x = 0 is
(A) $\frac{1}{2}$ (B) 2 (C) 1 (D) 0

Q.21 The product of x^2y and $\frac{x}{y}$ is equal to the quotient obtained when x^2 is divided by ______.
(A) 0 (B) 1 (C) x (D) $\frac{1}{x}$

Q.22 If $(3x - 4)(5x + 7) = 15x^2 - ax - 28$ then a = ______ .
(A) 1 (B) –1 (C) –2 (D) none

Q.23 One of the factors of $x^4 + 4$ is
(A) $x^2 + 2$ (B) $x^2 - 2x + 2$
(C) $x^2 - 2$ (D) none of these

Q.24 Which of the following expressions is a polynomial ?
(A) $3x^{\frac{1}{2}} - 4x + 3$ (B) $4x^2 - 3\sqrt{x} + 5$
(C) $3x^2y - 2xy + 5x^4$ (D) $2x^4 + \frac{3}{x^2} - 1$

Q.25 The degree of the polynomial $5x^3 - 6x^3y + 4y^2 - 8$ is
(A) 3 (B) 4
(C) 2 (D) can't be determined

Q.26 The factors of $x^4 + 2x^2 + 9$ is
(A) $(x^2 - 2x + 3)(x^2 + 2x + 3)$
(B) $(x^2 + 3)(x^2 - 3)$
(C) factorization is not possible
(D) none of these

Q.27 For $x^2 + 2x + 5$ to be a factor of $x^4 + px^2 + q$, the values of p and q must be
(A) −2, 5 (B) 5, 25 (C) 10, 20 (D) 6, 25

Q.28 The factors of $x^2 + xy - 2xz - 2yz$ are
(A) $(x - y)(x + 2z)$ (B) $(x + y)(x - 2z)$
(C) $(x - y)(x - 2z)$ (D) $(x + y)(x + 2z)$

Q.29 $x^9 - x$ is having
(A) 5 factors (B) 4 factors
(C) 2 factors
(D) cannot be determined

Q.30 The factors of $\frac{x^2}{4} - \frac{y^2}{9}$ are
(A) $\left(\frac{x}{4} + \frac{y}{9}\right)\left(\frac{x}{4} - \frac{y}{9}\right)$ (B) $\left(\frac{x}{2} + \frac{y}{9}\right)\left(\frac{x}{2} - \frac{y}{9}\right)$
(C) $\left(\frac{x}{2} + \frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)$ (D) none of these

Q.31 The factors of $1 - p^3$ are
(A) $(1 - p)(1 + p + p^2)$
(B) $(1 + p)(1 - p - p^2)$
(C) $(1 + p)(1 + p^2)$
(D) $(1 + p)(1 - p^2)$

Q.32 The value of
$\frac{0.76 \times 0.76 \times 0.76 + 0.24 \times 0.24 \times 0.24}{0.76 \times 0.76 - 0.76 \times 0.24 + 0.24 \times 0.24}$ is
(A) 0.52 (B) 1 (C) 0.01 (D) 0.1

Q.33 The factors of $a^2 + b - ab - a$ are
(A) $(a - 1)(a - b)$ (B) $(a + 1)(a - b)$
(C) $(a - 1)(a + b)$ (D) $(a + b)(a - b)$

Q.34 One of factors of $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$ is
(A) $x - \frac{1}{x}$ (B) $x + \frac{1}{x} - 1$
(C) $x + \frac{1}{x}$ (D) $x^2 + \frac{1}{x^2}$

Q.35 If the factors of $a^2 + b^2 + 2(ab + bc + ca)$ are $(a + b + m)$ and $(a + b + nc)$, then the value of $m + n$ is
(A) 0 (B) 2 (C) 4 (D) 6

Q.36 If $(x^2 + 3x + 5)(x^2 - 3x + 5) = m^2 - n^2$, them m = ______
(A) $x^2 - 3x$ (B) $3x$ (C) $x^2 + 5$ (D) none

Q.37 One of the factors of $(a^2 - b^2)(c^2 - d^2) - 4abcd$ is
(A) $(ac - bd + bc + ad)$
(B) $ac - bd + bc - ad$
(C) cannot be determined
(D) none of these

Q.38 The factors of $\sqrt{3}x^2 + 11x + 6\sqrt{3}$ are
(A) $(x - 3\sqrt{3})(\sqrt{3}x + 2)$
(B) $(x - 3\sqrt{3})(\sqrt{3}x - 2)$
(C) $(x + 3\sqrt{3})(\sqrt{3}x - 2)$
(D) $(x + 3\sqrt{3})(\sqrt{3}x + 2)$

Q.39 The difference of the degree of the polynomials $3x^2y^3 + 5xy^7 - x^6$ and $3x^5 - 4x^3 + 2$ is
(A) 2 (B) 3 (C) 1 (D) none

Q.40 What must be added to $x^2 + 5x - 6$ to get $x^3 - x^2 + 3x - 2$?
(A) $x^3 - 2x^2 - 2x - 4$ (B) $x^3 + 2x^2 - 2x + 4$
(C) $x^3 - 2x^2 - 2x + 4$ (D) none

Q.41 What must be subtracted from $x^4 + 2x^2 - 3x + 7$ to get $x^3 + x^2 + x - 1$?
(A) $x^4 - x^3 + x^2 - 4x + 8$
(B) $x^3 + x^2 - 4x + 8$
(C) $x^4 - x^3 + x^2 + 4x - 8$
(D) $x^4 - x^3 - x^2 + 4x - 8$

Q.42 One of the factors of $x^7 + xy^6$ is
(A) $x^2 + y^2$ (B) x
(C) either A or B (D) neither A nor B

Q.43 Factors of $x^3 + x^2 + x + 1$
(A) $(x + 1)(x^2 - 1)$ (B) $(x - 1)(x^2 + 1)$
(C) $(x - 1)(x^2 - 1)$ (D) $(x + 1)(x^2 + 1)$

Q.44 Factors of $x^2 + (a + b + c)x + ab + bc$
(A) $(x + a)(x + b + c)$ (B) $(x + a)(x + a + c)$
(C) $(x + b)(x + a + c)$ (D) $(x + b)(x + b + c)$

Q.45 Factorise $x^{12} - y^{12}$
(A) $(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
(B) $(x + y)(x^2 - xy + y^2)(x + y)(x^2 - xy + y^2)(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
(C) $(x + y)(x^2 + xy - y^2)(x + y)(x^2 - xy + y^2)(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
(D) $(x - y)(x^2 - xy + y^2)(x + y)(x^2 - xy + y^2)(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

Q.46 If $x + y + z = 0$ then $x^3 + y^3 + z^3$ is
(A) xyz (B) 2xyz (C) 3xyz (D) zero

Q.47 If $a + b + c = 0$, evalute $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$
(A) 1 (B) 2 (C) 3 (D) 4

Q.48 A factor of $x^3 - 1$ is
(A) $x - 1$ (B) $x^2 + x + 1$
(C) either A or B (D) none of these

Q.49 The product of $\frac{2}{3}xy$ by $\frac{3}{2}xz$ is
(A) $\frac{1}{6}xyz$ (B) x^2yz
(C) $6x^2yz$ (D) none of these

Q.50 Divide $8x^2y^2 - 6xy^2 + 10x^2y^3$ by $2xy$.
(A) $4xy - 3y + 5xy^2$ (B) $4xy + 3y - 5xy^2$
(C) $8xy + 3y - 5xy^2$ (D) $4xy - 3y - 5xy^2$

Q.51 Factors of $ab + bc + ax + cx$
(A) $(a + c)(x + b)$ (B) $(a - c)(x - b)$
(C) $(a + b)(x + c)$ (D) $(x + a)(a + b)$

Q.52 Factors of $20a^2 - 45$
(A) $5(3 - 2a)(3 + 2a)$ (B) $5(2a + 3)(2a - 3)$
(C) $3(5 + 2a)(5 - 2a)$ (D) $3(2a + 5)(2a - 5)$

Q.53 Resolve into factors. $6x^3 - 24xy^2 - 3x^2y + 12y^3$
(A) $3(2x - y)(x + 2y)(x - 2y)$
(B) $3(2x - y)(x + y)(x + 2y)$
(C) $3(2x - y)(x - 2y)(x + y)$
(D) $3(2x + y)(x - y)(x + 2y)$

Q.54 If the polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by $x - 2$ then the value of a is
(Hint : Substitute $x = 2$ in both polynomials and equate.)
(A) $\frac{3}{13}$ (B) $\frac{3}{14}$ (C) $-\frac{13}{3}$ (D) $-\frac{3}{13}$

Q.55 If R_1 and R_2 are remainder when $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ and if $2R_1 + R_2 = 6$ then the value of a is
(A) 1 (B) 2 (C) 3 (D) 4

Q.56 If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$ then the condition is
(A) $a + c + e = b + d$ (B) $a - c - e = b - d$
(C) $a + c - e = b - d$ (D) $a + c + e = d - b$

Q.57 Factorise $1 - x + x^2 - x^3$
(A) $(1 + x)(1 - x^2)$ (B) $(1 - x)(1 + x^2)$
(C) $(1 - x)(1 - x^2)$ (D) $(1 + x)(1 + x^2)$

Q.58 Factorise $49y^2 - 14y + 1 - 25x^2$
(A) $(7y - 1 + 5x)(7y - 1 - 5x)$
(B) $(5y - 1 + 7x)(5x - 1 - 7x)$
(C) $(7y - 1 + 5x)(7y - 1 + 5x)$
(D) $(5x + 7y - 1)(5x + 7y + 1)$

Q.59 The value of $2^{\frac{1}{4}} \cdot 4^{\frac{1}{8}} \cdot 16^{\frac{1}{16}} \cdot (256)^{\frac{1}{32}}$ is equal to
(A) 1 (B) 2 (C) 4 (D) 8

Q.60 Factors of $x - 8xy^3$
(A) $x(1 - 2y)(1 + 2y + 4y^2)$
(B) $x(1 + 2y)(1 + 2y + 4y^2)$
(C) $x(1 - 2y)(1 - 2y + 4y^2)$
(D) $x(1 + 2y)(1 - 2y + 4y^2)$

Q.61 Factors of $(m - n)^6 - 8m^3$
(A) $\{(m-n)^3 + 4m\}\{(m-n)^3 - 2m^2\}$
(B) $\{(m-n)^3 + (2m)^3\}\{(m-a)^3 + (m^2)\}$
(C) $\{(m-n)^2 - 2m\}\{(m-n)^4 + (m-n^2)2m + 4m^2)\}$
(D) none of those

Q.62 If $a^{\frac{1}{2}} + b^{\frac{1}{2}} - c^{\frac{1}{2}} = 0$ then the value of $(a + b - c)^2$ is
(A) 2ab (B) 2bc (C) 4ab (D) 4ac

Q.63 Factorise $a^{2x} - b^{2x}$
(A) $(a^x + b^x)(a^x - b^x)$ (B) $(a^x - b^x)^2$
(C) $(a^x + b^x)(a^2 - b^2)$ (D) $(a^x - b^x)(a^2 + b^2)$

Q.64 If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$, $z = \frac{c-a}{c+a}$ then the value of $\frac{(1+x)(1+y)(1+z)}{(1-x)(1-y)(1-z)}$ is
(A) abc (B) $a^2b^2c^2$ (C) 1 (D) −1

Q.65 If $11^7 + 4^7$ is divided by 15 then the remainder is
(A) 0 (B) 1 (C) 2 (D) −2

Q.66 If $5^{2n} - 2^{3n}$ is divisible by 17 then the remainder is
(A) 0 (B) 1 (C) –1 (D) 2

Q.67 The value of $25x^2 + 16y^2 + 40xy$ at $x = 1$ and $y = -1$ is
(A) 81 (B) –49 (C) 1 (D) none

Q.68 Factorise $8a^3 - 2a^2 - 15ab^2$
(A) $(4a + 5b)(2a^2 - 3ab)$
(B) $(4a - 5b)(2a^2 - 3ab)$
(C) $(4a - 5b)(2a^2 + 3ab)$
(D) $(4a + 5b)(2a^2 + 3ab)$

Q.69 If $p = (2 - a)$ then $a^3 + 6ap + p^3 - 8$ is
(A) 0 (B) 1 (C) 2 (D) 3

Q.70 The value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ when $a + b + c = 3x$
(A) 3 (B) 2 (C) 1 (D) 0

Q.71 The remainder when $P(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$ is divided by $x - 1$ is
(A) –7 (B) –6 (C) 7 (D) 6

Q.72 Factors of $x^3 + 7x^2 + 14x + 8$
(A) $(x - 1)(x - 2)(x - 4)$
(B) $(x + 1)(x + 2)(x - 4)$
(C) $(x + 1)(x - 2)(x - 4)$
(D) $(x + 1)(x + 2)(x + 4)$

Q.73 $f(x) = ax^7 + bx^3 + cx - 5$ where a, b, c are constants. If $f(-7) = 7$ then $f(7)$ equals to [Hint : Find $f(-x)$, and find the sum of $f(-x)$ and $f(x)$]
(A) –17 (B) –7 (C) 14 (D) 21

Q.74 Factors of $\frac{1}{2}x^2 - 3x + 4$
(A) $\left(\frac{1}{4}x - 1\right)(x - 2)$ (B) $(x - 4)\left(\frac{1}{2}x - 1\right)$
(C) $(x - 4)(x - 2)$ (D) $(x - 4)(x - 1)$

Q.75 Factorise $(3 - 4y - 7y^2)^2 - (4y + 1)^2$
(A) $(4 - 7y^2)(2 - 8y - 7y^2)$
(B) $(7y^2 - 4)(2 - 8y - 7y^2)$
(C) $(4 - 7y^2)(7y^2 + 8y - 2)$
(D) $(7y^2 - 4)(7y^2 - 8y - 2)$

Q.76 Factorise $3\sqrt{3}x^3 + y^3$
(A) $(\sqrt{3}x + y)(3x^2 - \sqrt{3}xy + y^2)$
(B) $(\sqrt{3}x - y)(\sqrt{3}x^2 + \sqrt{3}xy + y^2)$
(C) $(\sqrt{3}x + y)(3x^2 + \sqrt{3}xy + y^2)$
(D) $(\sqrt{3}x - y)(3x^2 - \sqrt{3}xy + y^2)$

Q.77 If $3x - 7y = 10$ and $xy = -1$ then the value of $9x^2 + 49y^2$ is
(A) 58 (B) 142 (C) 104 (D) –104

Q.78 If $a + b + c = 10$ and $a^2 + b^2 + c^2 = 36$ then $ab + bc + ca =$ _____
(A) 136 (B) 64 (C) 32 (D) 68

Q.79 If $x - y = 4$ and $xy = 21$ then $x^3 - y^3 =$
(A) 361 (B) 316 (C) –188 (D) none

Q.80 Find the value of a if the polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x - a$ leave the same remainder when divided by $x - 1$.
(A) a = –1 (B) a = 1 (C) a = 2 (D) a = – 2

Q.81 The value of P if $(x - 3)$ is a factor of $p^2x^3 - px^2 + 3px - p$ is
(A) 27 (B) –27 (C) $\frac{1}{27}$ (D) $-\frac{1}{27}$

Q.82 Factorise $x^2 - 1 - 2a - a^2$
(A) $(x - a - 1)(x + a - 1)$
(B) $(x + a + 1)(x - a - 1)$
(C) $(x + a + 1)(x - a + 1)$
(D) $(x - a + 1)(x + a - 1)$

Q.83 Factors of $x^4 - x^2 - 12$
(A) $(x + 2)(x - 2)(x^2 + 3)$
(B) $(x + 3)(x - 3)(x^2 + 2)$
(C) $(x + 2)(x - 2)(x^2 - 3)$
(D) $(x^2 + 2)(x^2 - 6)$

Q.84 Is $(x - 1)$ a factor of $x^3 - 6x^2 + 11x - 6$?
(A) Yes (B) No
(C) Can't say (D) None

Q.85 The remainder when $p(x) = x^4 + 2x^3 - 3x^2 + x - 1$ is divided by $x - 2$ is
(A) 21 (B) –21 (C) 47 (D) –47

Q.86 The remainder when $f(x) = 2x^4 - 6x^3 + 2x^3 - x + 2$ is divided by $g(x) = x + 2$ is
(A) 29 (B) –29 (C) 92 (D) –92

Q.87 What should be added to $8x^3 - 4x^2 + 2x + 7$ so as to make the sum $10x^3 - 2x^2 + 7x + 20$?
(A) $-2x^3 + 2x^2 + 5x + 3$
(B) $2x^3 + 2x^2 + 5x + 3$
(C) $-2x^3 - 2x^2 - 5x + 13$
(D) $2x^3 + 2x^2 + 5x - 13$

Q.88 If $p(x) = 4x^3 - 3x^2 + 2x + 1$, $q(x) = x^3 - x^2 + x + 1$, $r(x) = x^2 - 2x + 1$ then the value of $3p(x) + 7q(x) + r(x)$ is
(A) $19x^3 - 15x^2 + 11x + 11$
(B) $-19x^3 - 15x^2 + 11x - 11$
(C) $19x^3 - 15x^2 - 11x + 11$
(D) $19x^3 - 15x^2 - 11x - 11$

Q.89 The value of m if $2y^3 + my^2 + 11y + m + 3$ is exactly divisible by $2y - 1$ is
(A) 7 (B) −7 (C) 6 (D) −6

Q.90 The value of m if $(x - 2)$ is a factor of $2x^3 - 5x^2 + 5x + m$
(A) −1 (B) 0 (C) 2 (D) −2

Q.91 If $x^3 + y^3 + z^3 = 3xyz$ then the relation between x, y and z is
(A) $x + y + z = 0$ (B) $x = y = z$
(C) either $x + y + z = 0$ (or) $x = y = z$
(D) neither $x + y + z = 0$ nor $x = y = z$

Q.92 The equality $b^2 + 5 > 9b + 12$ is satisfied if
(A) $b > 9$ (or) $b < 1$ (B) $b > 9$ (or) $b < 0$
(C) $b = 10$ (or) $b = -1$ (D) $b > 8$ (or) $b < 0$

Q.93 If $0 < a < 1$, then the value of $a + \frac{1}{a}$ is
(A) greater than 2 (B) less than 2
(C) greater than 4 (D) less than 4

Q.94 What is the zero of the binomial $ax + b$?
(A) 0 (B) $\frac{b}{a}$ (C) $\frac{-a}{b}$ (D) $\frac{-b}{a}$

Q.95 If $p(x, y) = a^2 - y^2 - xy$ and $q(x, y) = -x^2 + y^2 + 3xy$ then the value of $4p(x, y) - 5q(x, y)$ is
(A) $9x^2 - 9y^2 - 19xy$ (B) $9x^2 + 9y^2 - 19xy$
(C) $-9x^2 + 9y^2 + 19xy$ (D) $9x^2 - 9y^2 + 19xy$

Q.96 The value of a if $(x - a)$ is a factor of $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$
(A) $a = -1$ (B) $a = 2$ (C) $a = -2$ (D) $a = 1$

Q.97 Using factor therem, factorise the cubic polynomial $x^3 - 6x^2 + 11x - 6$.
(A) $(x + 1)(x - 3)(x - 2)$
(B) $(x - 1)(x - 3)(x - 2)$
(C) $(x + 1)(x + 3)(x - 2)$
(D) $(x + 1)(x - 3)(x + 2)$

Q.98 The value of $\frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$ is
(A) 9 (B) 6.66 (C) 1.176 (D) none

Q.99 If $x - \frac{1}{x} = \sqrt{6}$ then $x^2 + \frac{1}{x^2} =$ _____
(A) 2 (B) 4 (C) 6 (D) 8

Q.100 Factorise $x^3 - 2x^2 - x + 2$.
(A) $(x - 1)(x - 2)(x + 1)$
(B) $(x + 1)(x + 2)(x - 1)$
(C) $(x - 2)(x - 1)^2$
(D) $(x + 2)(x - 1)(x - 2)$

Q.101 When $f(x)$ is divided by $2x + 3$ then the remainder is ?
(A) $f(2)$ (B) $f(3)$ (C) $f(3/2)$ (D) $f(-3/2)$

Q.102 Factors of $a^3 + b^3 + a + b$
(A) $(a + b)(a^2 + b^2 - ab + 1)$
(B) $(a - b)(a^2 + b^2 - ab + 1)$
(C) $(a + b)(a^2 - b^2 + ab - 1)$
(D) $(a + b)(a^2 - ab - b^2 - 1)$

Q.103 For what value of K is the polynomial $2x^4 + 3x^3 + 2kx^2 + 3x + 6$ is exactly divisible by $(x + 2)$?
(A) $k = 1$ (B) $k = -1$ (C) $k = 2$ (D) $k = -2$

Q.104 When $x^{11} + 1$ is divided by $x + 1$ then the remainder is
(A) 0 (B) 2 (C) 1 (D) −1

Q.105 When $x^9 - a^9$ is divided by $x - a$ then the remainder is
(A) 0 (B) 1 (C) −1 (D) 2

Q.106 What must be substracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the result is exactly divisible by $2x^2 + x - 2$?
(A) $-3x - 5$ (B) $3x - 5$ (C) $-3x + 5$ (D) $3x - 5$

Q.107 If $(x + 1)$ and $(x - 1)$ are the factors of $px^3 + x^2 - 2x + q$ then the values of p and q are
(A) $p = -1, q = 2$ (B) $p = 2, q = -1$
(C) $p = 2, q = 1$ (D) $p = -2, q = -2$

Q.108 Find the missing term in the following problem.

$$\left(\frac{3x}{4} - \frac{4y}{3}\right)^2 = \frac{9x^2}{16} + \_\_\_\_\_ + \frac{16y^2}{9}$$

(A) 2xy (B) −2xy (C) 12xy (D) −12xy

Q.109 $x^2 + \frac{1}{x^2} = 79$ then $x + \frac{1}{x}$ = _____

(A) $\sqrt{75}$ (B) 9

(C) $\sqrt{79}$ (D) none

Q.110 Factorise $px^2 + (4p^2 - 3q)x - 12pq$

(A) $(x - 4p)(px - 3q)$

(B) $(x + 4p)(px - 3q)$

(C) $(x + 4p)(px + 3q)$

(D) $(x - 4p)(px + 3q)$

Q.111 If $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$ then

(A) $x^3 + y^3 + z^3 = 0$

(B) $x + y + z = 27xyz$

(C) $(x + y + z)^3 = 27xyz$

(D) $x^3 + y^3 + z^3 = 27xyz$

Q.112 $(a + b)^3 - (a - b)^3$ =

(A) $b^3 + 3a^2b$ (B) $2(b^3 + 2a^2b)$

(C) $2(a^3 + 3ab^2)$ (D) 0

Q.113 The product of $(4x - 3y)$ and $(16x^2 + 12xy + 9y^2)$

(A) $(4x - 3y)^3$

(B) $(16x^2 + 12xy + 9y^2)^2$

(C) $64x^3 - 27y^3$

(D) none

Q.114 $(3x - 5y)^3 - (5x - 2y)^3 + (2x + 3y)^3$

(A) $(3x - 5y)(5x - 2y)(2x + 3y)$

(B) $3(3x - 5y)(5x - 2y)(2x + 3y)$

(C) $(3x - 5y)(2y - 5x)(2x + 3y)$

(D) $-3(3x - 5y)(5x - 2y)(2x + 3y)$

Q.115 If $a + b + c = 9$ and $ab + bc + ca = 26$, then the value of $a^3 + b^3 + c^3 - 3abc$ is

(A) 27 (B) 29

(C) 495 (D) 729

ANSWER KEY

1.	A	**2.**	A	**3.**	C	**4.**	B
5.	C	**6.**	C	**7.**	D	**8.**	C
9.	B	**10.**	D	**11.**	A	**12.**	A
13.	B	**14.**	D	**15.**	A	**16.**	A
17.	B	**18.**	A	**19.**	C	**20.**	D
21.	D	**22.**	B	**23.**	B	**24.**	C
25.	B	**26.**	A	**27.**	D	**28.**	B
29.	A	**30.**	C	**31.**	A	**32.**	B
33.	A	**34.**	C	**35.**	B	**36.**	C
37.	A	**38.**	D	**39.**	B	**40.**	C
41.	A	**42.**	C	**43.**	D	**44.**	C
45.	A	**46.**	C	**47.**	C	**48.**	C
49.	B	**50.**	A	**51.**	A	**52.**	B
53.	A	**54.**	C	**55.**	B	**56.**	A
57.	B	**58.**	A	**59.**	B	**60.**	A
61.	C	**62.**	C	**63.**	A	**64.**	C
65.	A	**66.**	A	**67.**	C	**68.**	A
69.	A	**70.**	D	**71.**	A	**72.**	D
73.	A	**74.**	B	**75.**	A	**76.**	A
77.	A	**78.**	C	**79.**	B	**80.**	A
81.	C	**82.**	B	**83.**	A	**84.**	A
85.	A	**86.**	C	**87.**	B	**88.**	A
89.	B	**90.**	D	**91.**	C	**92.**	B
93.	A	**94.**	D	**95.**	A	**96.**	A
97.	B	**98.**	A	**99.**	D	**100.**	A
101.	D	**102.**	A	**103.**	B	**104.**	A
105.	A	**106.**	A	**107.**	B	**108.**	B
109.	B	**110.**	B	**111.**	C	**112.**	B
113.	C	**114.**	D	**115.**	A		